DESIGN OF LOW NOISE HIGH POWER RF AMPLIFIER USING BIPOLAR JUNCTION TRANSISTORS

A PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
in
Electronic Systems and Communication Engineering

By

BHANJA KISHOR SWAIN

Department of Electrical Engineering

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Under The Guidance of

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CERTIFICATE

This is to certify that the project entitled, " **Design of Low Noise High Power RF Amplifier Using Bipolar Junction Transistors"** submitted by **Sri Bhanja Kishor Swain** In partial fulfillments for the requirements for the award of Master of Technology Degree in Electrical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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Bhanja Kishor Swain
M.Tech (Electronic Systems and communication Engineering)

Contents

Abstract

The radio frequency amplifiers differ from audio frequency amplifiers in the choice of values and the circuit elements. In most applications of the microwave amplifiers not only high amplification is desirable, but the usable bandwidth should be as great as possible. The ordinary amplifiers can not operate at microwave ranges because of their inherent parasitic parameters and thus, it is necessary to design a Microwave Amplifiers, which is free from above bottlenecks.

This work concerns with the design of low noise microwave amplifier using Transistors. Microwave transistors are essentially scaled-down version of low frequency transistors. They find applications as local oscillators for radars and as radio frequency sources for low power transmitters apart from low noise microwave amplifiers .Most microwave bipolar transistors are generally Silicon n-p-n devices.

The design objective in this work is of two fold:

1) Determination of scattering parameters from a given set of power gains and noise figure .The design is ,however ,based on a single operating frequency .This, of course, can function well within a small band of frequencies with a centre frequency equals to the operating frequency.

2) Determination of performance index of a microwave amplifier, that is, Noise Figure(F),Gain(gp),Voltage standing wave ratio at input and at out put.

Also the detail calculations are presented in chapter-3; it is worthwhile to mention that the design problem undertaken satisfies the need of an amplifier, which can be used for microwave applications.

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List of Tables

Legends

b_1	Reflected power wave at port 1
b_2	Transmitted power wave at port 2
a_1	Incident power wave at port 1
a_2	Transmitted power wave at port 2
S_{11}, S_{21}, S_{22}, S_{12},	Scattering parameters
G_p	Power gain of the amplifier
G_t	Transducer gain of the amplifier
G_a	Available gain of the amplifier
G_{max}	Maximum gain of the amplifier
G_p	Maximum power gain
P_{inc}	Incident power gain
P_{in}	Input power
Y_c	Characteristic admittance
Γ	Reflection coefficient
P_L	Power delivered to the output
Gp_1	Power gain at stage 1
Gp_2	Power gain at stage 2
M	Impedance mismatch
Z_s	Amplifier source impedance
Z_{in}	Amplifier input impedance
Z	Characteristic impedance
Z_L	Load impedance
Γ_L	Load reflection coefficient
Γ_s	Source reflection coefficient
Γ_{in}	Input reflection coefficient
Γ_{out}	Output reflection coefficient
Γ_m	Minimum reflection coefficient
V_L	Load voltage
V_S	Source voltage
I_S	Source Current
I_L	Load current

M_s	Impedance mismatch at source end
M_L	Impedance mismatch at load end
P_{ava}	Available power
K	Stability parameter
Δ	Determinant of the scattering matrix
Γ_{LC}	Centre of the mapped circle $\lvert\Gamma_L\rvert$
R_{LC}	Radius of the mapped circle $\lvert\Gamma_L\rvert$
Γ_{SC}	Centre of the mapped circle $\lvert\Gamma_S\rvert$
R_{SC}	Radius of the mapped circle $\lvert\Gamma_S\rvert$
Γ_{LG}	Centre of the circle defined by g_p on Γ_L plane
R_{LG}	Radius of the circle defined by g_p on Γ_L plane
$e_n(t)$	Noise voltage
$C(\tau)$	Correlation function
$S_n(w)$	Power spectral density of noise
G_i	Noise conductance
R_e	Noise resistance
F	Noise figure
g_p	Normalized power gain
F_m	Minimum noise figure
G_p	Maximum power gain
Γ_{sf}	Centre of the constant noise figure circle
R_f	Radius of the constant noise figure circle
$VSWR_{in}$	Input voltage standing wave ratio
$VSWR_{out}$	Output voltage standing wave ratio

Chapter 1

INTRODUCTION

1.1 INTRODUCTION

Semiconductor devices have been in use for last few decades in microware applications. However, their extension to higher and higher frequencies has been rather slow. In the lower frequency regions, vacuum tubes are being gradually replaced by a variety of solid-state devices. However, until recently, the frequency range above one GHz remained the preserve of vacuum tubes. Microwave semiconductor devices and their associative circuits can perform a variety of functions, such as, generation, amplification, detection, switching modulation and limitation. Harmonics of the microwave signals can be generated; mixing of two microwave signals can be done and so on. In each function, the design of device and circuit is different.

The first solid-state amplifiers for microwave applications were negative resistance diodes, for example, tunnel diode. This was followed by the development of parametric amplifiers that used as variable capacitance diode (Varactor) and an oscillator to vary the junction capacitance. The main feature of parametric amplifiers was the low noise that could be achieved by cooling the diode to liquid nitrogen temperatures. Power outputs greater than 10 mw for tunnel diodes were difficult to obtain. On the order hand, the varactor multiplier source was more successful, since it uses a high power high frequency generating power at a few hundred megahertz. Here, the transistor drives a nonlinear reactance, harmonics are generated, and the power transfer from the fundamental to the harmonics can be quite efficient.

Parametric amplifiers became the prominent and most widely used solid-state amplifiers during the period 1958 to about 1970. By 1970, improvements in materials preparation and processing technology had resulted in development of n-p-n Silicon bipolar transistors with a maximum frequency of oscillation greater than 10 GHz. During the next two decades further progress in the design and manufacture of high frequency microwave bipolar transistors and field- effect transistors was dramatic. The key to successful microwave transistors design is miniaturization, which is a necessity in order to reduce device and package parasitic capacitances, lead inductances, as also to overcome the lead inductances and the finite transit time of the charge carriers. An appreciation for the need to reduce parasitic capacitances and inductance can be obtained by referring to the table.1.1.

Table 1.1: Reactance as a function of frequency

Frequency (GHz)	1	10	100
Reactance			
L=0.1 nH	0.628	6.28	62.8
L=1nH	6.28	62.8	628
C=0.1 pf	1592	159	15.9
C= 1 pf	159	15.9	1.6

For example, as may be seen from table 1.1, an inductance of 0.1 nH at 10GHz represents a reactance of $6.28\,\Omega$ which is not negligible value in a 50 Ω system. Similarly, a capacitance of 0.1 pF at 10 GHz has a reactance of $159\,\Omega$ and would be a significant shunt reactance across a $50\,\Omega$ transmission line.

Transit times are dependent on the election mobility and saturation velocity in the semiconductor material. In this regards, Gallium Arsenide (GaAs) is better than silicon for high frequency devices. By 1890, the design and fabrication of metal semiconductor field effect transistor (MESFET) were well established in the frequency range above 5 GHz and MESFET devices are widely used.

In order to achieve the high frequency performance in transistors, it was necessary to develop the technology that would enable key device dimensions to be less than 1 μm, for example, gate lengths with submicron dimensions.

1.2 MICROWAVE SEMICONDUCTOR DEVICES

There are mainly two types of devices used for the design of microwave amplifiers.

- Bipolar Junction Transistor
- Field Effect Transistors.

1.2.1 Microwave Bipolar Transistor

Microwave bipolar transistor are similar to low frequency transistors but are fabricated so as to have improved performance characteristics, such as, smaller transit time. To make the transit time a small fraction of a cycle, the base layer must be thin. At the same time, the base resistance must be small so that the inner collector capacitance can have a charging time, which is almost the same as the period of the operating frequency. The base doping cannot be made too high because, and then the efficiency with which the emitter injects minority carriers will be lowered. Hence, the charging time can be kept low only by making diameter narrow. Therefore, microwave transistors are now made in the form of many base and emitter strips very narrow and close to each other.

The collector base depletion layer is made thin so that the transit time from the base across the depletion layer to the collector is small. The emitter charging time is also made very low short. In the frequency range below 5GHz Silicon bipolar transistors are preferred except for very low noise amplifies design.

1.2.2 Field-Effect Transistors

Through field-effect transistors, including metal insulator semiconductor (MIS) devices, have a large number of uses, their speed at microwave frequency is not very high. Hence, they are inferior to bipolar transistors. There are two main characteristics of field effect transistors that make them superior to bipolar transistor in microwave amplifiers. Theses are the lower noise characteristics and the higher frequency of operation (due to the higher electron mobility in GaAs). The higher electron mobility and the absence of shot noise that result in low noise.

1.2.3 Molecular Beam Epitaxy Technique Used in Manufacture of High Electron Mobility Transistor

By means of molecular beam epitaxy (MBE), it has been possible to grow high-quality epitaxial layers and controlled doping profiles in highly localized regions. MBE techniques also led to the development of hetero structures, which in turn, led to the development of the high electron mobility transistor (HEMT), which can operate at frequency as high as 100 GHz.

1.2.4 Bipolar Technology Vs MOSFET Technology

The BJT has the advantage over the MOSFET of a much higher Transconductance (gm) at the same value of DC bias current. Thus, much higher value of voltage gain is realized in a bipolar transistor amplifier stage than in the corresponding MOSFET circuit. Also, bipolar transistor amplifiers have superior high-frequency performance than their MOS counterparts.

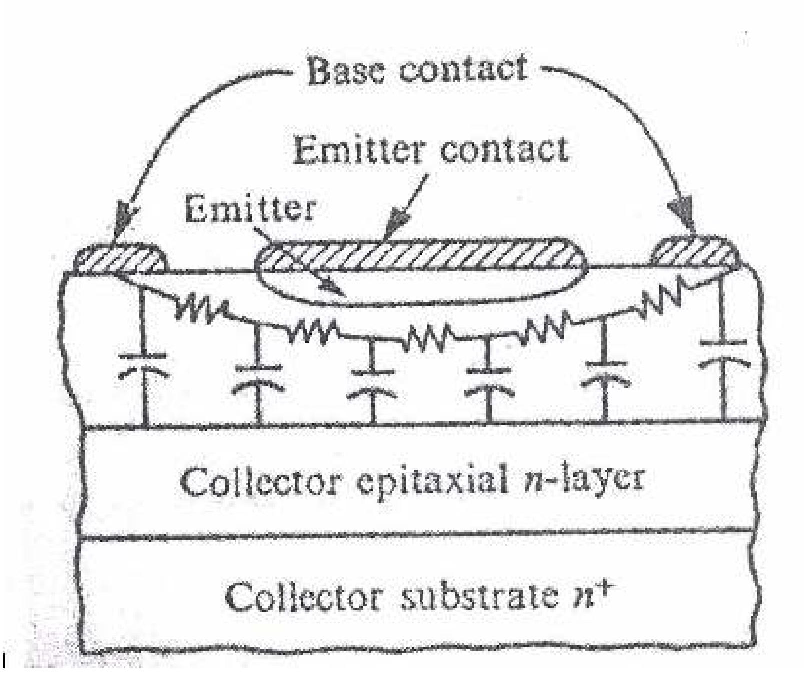


Fig.1.1 Double diffused epitaxial transistor

On the other hand, practically infinite input resistance at the gate of a MOSFET makes it possible to design MOS amplifiers with extremely high input resistance and an almost zero input bias current. Also, the MOSFET provides an excellent implementation of a switch: while a saturated BJT exhibits an offset voltage of few tenths of a volt, the I-V characteristics of the MOSFET pass right through the origin, resulting in zero offset. The availability of good switches in CMOS technology makes possible a host of analog circuit techniques that are employed in the design of, among other things data converters & filters.

It can, thus be seen that each of the two circuit technologies, Bipolar and CMOS, has its distinct and unique advantages.

1.3 MICROWAVE INTEGRATED CIRCUITS AND MONOLITHIC MICROWAVE INTEGRATED CIRCUITS

Microwave amplifiers usually constructed either as hybrid Microwave Integrated Circuits (MICs) or as Monolithic Microwave Integrated Circuits (MMICs). In hybrid construction, the transmission lines and matching networks are usually realized as micro strip circuit elements on a suitable substrate material and then the discrete components, such as, chip capacitor, resistors, and transistors are connected in place by soldering or using wire-bonding techniques. Discrete device are available with beam leads for easy insertion into the hybrid circuit.

A MMIC is a circuit where all active devices, for example transistors and passive circuit elements, such as transmission lines, capacitors, resistors and inductors are fabricated on a single semiconductor crystal. The substrate material used has typically been GaAs, because of its high resistivity in the undoped state.

In the frequency range below 10 GHz, where distributed circuit elements are relatively large, the hybrid form of construction is often less costly than monolithic construction. However, in the frequency range of 1.0 to 10 GHz, the ability to produce miniature inductors and capacitors has led to the development and production of many MMIC systems using lumped circuit elements instead of distributed circuit elements. The millimeter wavelength band monolithic microwave integrated circuit construction promises to be more cost effective and to yield circuits with greater reliability and uniformity.

1.4 NOISE IN AMPLIFIERS

It is found that there is inherent limit to the amplification obtainable from an amplifier, because even when there is no signal impressed at the input, a small output, called amplifier noise, is obtained. It therefore, only a very small voltage is available, such as, a weak radio, television, radar signal etc, it may be possible to distinguish the signal from the background noise.

The two important sources in an amplifier are:

(i) thermal noise

(ii) shot noise.

1.4.1 Thermal Noise

The electron in a conductor possesses varying amounts of energy by virtue of the temperature of the conductor. The slight fluctuations in energy account the values specified by the most probable distribution are very small, but they are sufficient to produce small noise potentials within a conductor. These random fluctuations produced by the thermal agitation of the electrons are called the thermal (or Johnson) noise.

1.4.2 Shot Noise

Shot noise is attributed to the discrete particle nature of current carriers in semiconductor. Naturally, one assumes that the current in a transistor or FET, DC conditions is a constant at every instant. Actually, however, the current from the emitter to the collector consists of a stream of individual electrons or holes, and it is only the time average flow, which is measured as the constant current. The fluctuation in the number of carriers is called shot (Schottky) noise. Apart from the above noises, at low frequencies, the noise varies approximately as $1/f$ and called excess, or flicker noise. The source of the noise is not clearly understood but is thought to be caused by the recombination and generation of carriers on the surface of the crystal. In intermediate frequencies the noise is independent of frequency. This white noise, (which gives the same noise per unit bandwidth anywhere in the spectrum), is caused by the bulk resistance of the semiconductor material and the statistical variation of the current (short noise) at a still higher frequencies noise increases and is essentially caused by a decrease in power gain with frequency.

1.5 SCATTERING MATRIX

The scattering matrix is a useful analytical technique for studying multiport microwave networks. Its elements relate forward and reverse traveling waves at the various ports of the network. The technique is a logical extension of th4 interpretation of transmission line phenomena in terms of incident and reflected waves. In classical circuit theory, the network is usually characterized by an impedance or admittance matrix that relates the terminal voltages and currents. An alternate method that is quite useful in microwave analysis is to describe the network behavior in terms of incident and scattered waves.

The incident and scattered waves are related to the network characteristics in matrix form by the following equations [1, 2]:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{12}a_1 + S_{22}a_2$$

In matrix form, $[b] = [S][a]$

Where [a] and [b] are column matrices, that represents the incident and scattered waves. The scattering coefficients (S_{11}, S_{12}, ..etc) define the characteristics of the linear network. The scattering matrix is symmetrical.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

The elements of the matrix are generally complex. These elements are easily measured from an actual network. For reciprocal networks, the scattering matrix is symmetrical.

1.6 BILINEAR TRANSFORMATION

Most of the relationships involved in amplifier design are of the form of a bilinear transformation from one complex variable to another complex variable.

If Z and W are the two complex variables, then an equation of the form [1]

$$w = \frac{AZ + B}{CZ + D} \qquad (1.1)$$

Where A, B, C and D are complex constants. The bilinear transformation has the property that circles in the Z plane will map into circles in the W plane with straight lines as limiting forms of circles, with infinite radii, and some points as circles with zero radius.

1.6.1 Circle-mapping properties of bilinear transformation

A circle with center at (x, y) and with radius R is described by

$$(x - x_0)^2 + (y - y_0)^2 - R^2 = 0$$

or, $x^2 + y^2 - 2xx_0 + 2yy_0 + (x_0^2 + y_0^2 + R^2) = 0$ (1.2)

Now let, $z = x + jy$, $z_0 = x_0 + jy_0$,

Then the above equation of the circle can be written as:

$$|z - z_0|^2 - R^2 = 0$$

or, $(z - z_0)(z^* - z_0^*) - R^2 = o$

or, $zz^* - zz_0^* - z^*z_0 + (z_0 z_0^* - R^2) = 0$ (1.3)

Let us now consider the bilinear transformation from the complex Z-plane to the complex W-plane given by

$$W = \frac{AZ + B}{CZ + D}$$

Where A, B, C and D are complex constants.

This transformation will map circles in the Z-plane into circles in the W-plane. Considering the circle $|W^2| = \rho^2$

Or, $WW^* - \rho^2 = 0$ (1.4)

Putting the value of W from Equation 1.4,

$$\frac{AZ + B + A^*Z^* + B^*}{CZ + DC^*Z^* + D^*} - \rho^2 = 0$$

By solving the equation,

$(AZ+B)(A^*Z^*+B^*)-\rho^2(CZ+D)(C^*Z^*+D^*)=0$

or,

$$AA^*ZZ^* + AZB^* + BA^*Z^* + BB^* - \rho^2 CC^*ZZ^* - \rho^2 CZD^* - \rho^2 DC^*Z^* - \rho^2 DD^* = 0$$

or, $ZZ^*\left(AA^* - \rho^2 CC^*\right) - Z\left(\rho^2 CD^* - AB^*\right) - Z^*\left(\rho^2 DC^* - BA^*\right) + BB^* - \rho^2 DD^* = 0$

By comparing with Equation (1.1), it is seen that this is circle with center at

$$Z_0 = \frac{\rho^2 C^* D - A^* B}{AA^* - \rho^2 CC^*} = \frac{\rho^2 C^* D - A^* B}{|A|^2 - \rho^2 |C|^2} \tag{1.5}$$

and the radius of the circle is

$$R = \rho \frac{|AD - BC|}{\left\| A|^2 - \rho^2 |C|^2 \right|}$$

1.7 ORGANIZATION OF THESIS

This thesis attempts at the design of low noise amplifier based on BJT. This thesis contains four chapters. These chapters are arranged to provide an introduction of microwave semiconductor electron devices followed by the basic needs to design a microwave amplifier. These form contents of chapter 1.

In chapter 2, the design methodology is presented in detail. The need of scattering matrix for such design is highlighted.

Chapter 3, deals with the results obtained using the design method presented in chapter 2 and discussion thereon.

Finally, the conclusions are presented in chapter 4, based on the information available in the literature while designing a LNA and work carried out in this thesis.

Chapter 2

DESIGN METHODOLOGY FOR MICROWAVE AMPLIFIER

2.1 DESIGN METHODOLOGY

In chapter 1, a brief introduction of amplifiers is presented in general and microwave amplifiers in particular. In this chapter, the design methodology used for microwave amplifiers is presented. The design of microwave amplifiers here is based on the engineering point of view, that is, starting from the measured or manufacture's given two-port parameters of the device, an amplifier is designed that meets a set of given system requirements, such as, gain, noise figure, band width and input & output Voltage Standing Wave Rations (VSWR). The design methodology adopted here is based on the use of the scattering matrix parameters for the device.

2.2 MICROWAVE AMPLIFIER DESIGN USING SCATTERING PARAMETER

. At microwave frequencies, impedance and admittance parameters of a transistor cannot be directly measured, whereas the scattering matrix parameters of a transistor can be measured easily. Therefore, a design methodology based on using the S_{ij} parameters is widely used. Many of the relationship that occurs in amplifier design involve S-parameters.

2.3 DESIGN GOALS

The following are the useful goals for designing microwave amplifier [8]:

1. Maximum power gain
2. Stable gain, that is, no oscillation.
3. Input and output VSWR as close to unity as possible.
4. Minimum noise figure.

However, all these objectives cannot be realized at the same time. So, the design procedure must trade off one objective against another. For example, gain must be sacrificed for stability, input VSWR must be sacrificed for a low noise figure and so on.

AMPLIFIER POWER GAIN

There are several definitions used for the gain of an amplifier and they are given as [1, 2]:

$$power\ \ gain, G_p = \frac{P\text{ower} \quad \text{delivered to load}}{\text{Input power to amplifier}}$$

Transducer gain, $G_t = \dfrac{Power\ \text{delivered to load}}{\text{Available input power from source}}$

Available power gains, $G_a = \dfrac{\text{Available load power}}{\text{Available input power from source}}$

Out of these, the power gain is the most useful definition in practice, since, it applies to any actual amplifier independent of whether conjugate impedance matching is used or not. If the device that is only conditionally stable, conjugate impedance matching at both the input and output cannot be used when the stability parameter, K<1.

Let us now consider a two-stage amplifier as shown figure 2.1.

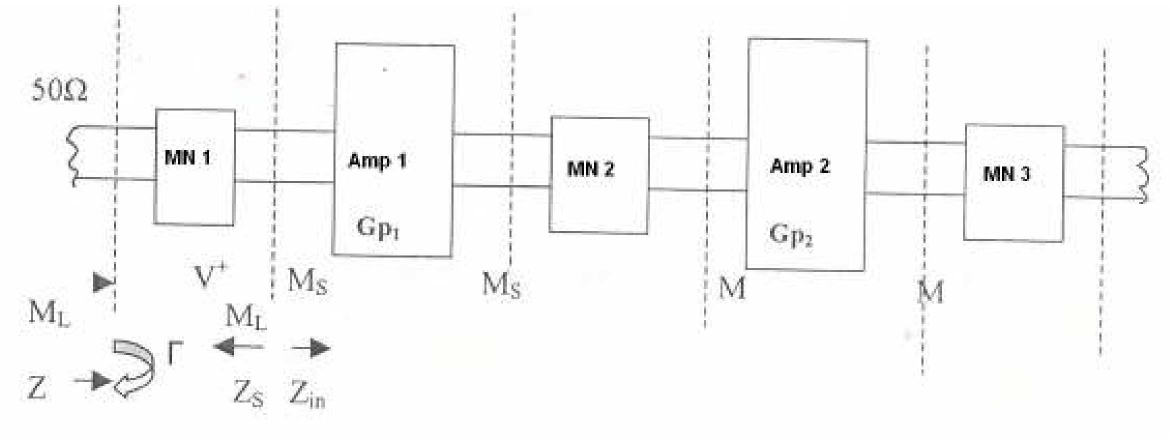


Figure 2.1: Two- stage Amplifier with Matching Networks

For the 2- stage amplifiers shown in figure 2.1, the incident power is given by [1]:

$$P_{inc} = \frac{1}{2}\left|V^{+}\right|^{2} Y_c$$

Where Y_c is the characteristic admittance of the amplifier.

The input power is

$$P_{in} = \left(1 - |\Gamma|^2\right) P_{inc}$$

Where Γ is the input reflection coefficient

Therefore, the power delivered to the output of 50 Ω line is

$$P_L = G_{p1} G_{p2} P_{in}$$

Hence, the 2-stage power gain is

$$G_p = \frac{P_L}{P_{in}} = G_{pl} G_{p2} \tag{2.1}$$

The corresponding transducer gain is

$$G_t = \frac{P_L}{P_{in}} = \left(1 - |\Gamma|^2\right) G_{pl} G_{p2} \tag{2.2}$$

For lossless matching networks the impedance mismatch is the same on the input side as on the output side as shown.

If M_s is the impedance mismatch between the first amplifier input and its source impedance, Z_s, then

$$M_s = \frac{4R_s R_m}{|Z_c + Z_m|^2} \tag{2.3}$$

Where $Z_m = R_m + jX_{in}$, is the amplifier input impedance. On the input line

$$M_s = \frac{4R_c R}{|Z_c + Z|^2} = \frac{4R}{|1+Z|^2}$$

Where Z= $(1-\Gamma)/(1+\Gamma)$

Now, using the relation

$$2R = Z + Z^* = \frac{1-\Gamma}{1+\Gamma} + \frac{1-\Gamma^*}{1+\Gamma^*}$$

and $1 + Z = \dfrac{2}{1+\Gamma}$

It may be found that,

$$M = 1 - \Gamma\Gamma^* = 1 - |\Gamma|^2 = M_s \tag{2.4}$$

The input VSWR is given by

$$\text{VSWR}_1 = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\sqrt{1-M}}{1-\sqrt{1-M}} = \frac{1+\sqrt{1-M_s}}{1-\sqrt{1-M_s}} \tag{2.5}$$

Thus, the degree of mismatch between Zs and the input to stage 1 determines the input VSWR.

Expression for Gain

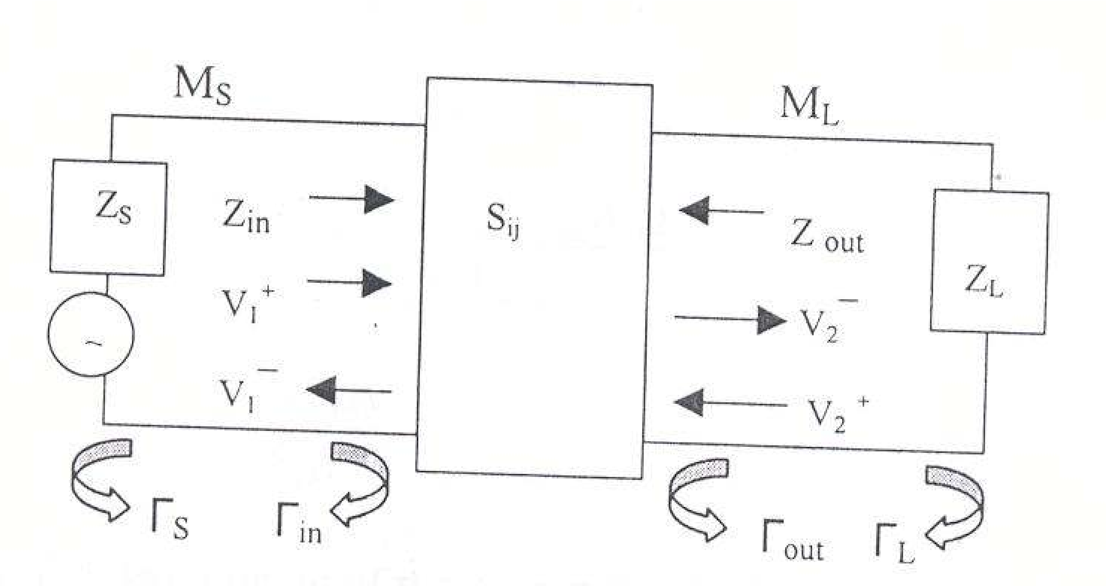


Fig.2.2 A basic Amplifier Circuit

For the basic amplifier circuit as shown in figure 2.2, the source and load are viewed as connected to the amplifier by means of transmission line with characteristic impedance Z_c and having negligible lengths. The source and load reflection coefficient for this circuit are given by

$$\Gamma_l = \frac{Z_L - 1}{Z_L + 1} \quad and \quad \Gamma_s = \frac{Z_s - 1}{Z_s + 1}$$

For the amplifier, the reflected voltages are given by

$$V_1^- = S_{11}V_1^* + S_{12}V_2^* \qquad (2.6a)$$

$$V_1^- = S_{21}V_1^* + S_{22}V_2^* \qquad (2.6b)$$

But $\quad V_2^* = \Gamma_L V_2^- \quad so \quad V_2^- = S_{21}V_1^* + S_{22}\Gamma_L V_2^-$

Or, $$V_2^- = S_{21}V_1^* \Big/ (1 - S_{22}\Gamma_L) \qquad (2.6c)$$

Putting the value of V^-_2 in equation (2.6a)

$$V_1^- - V_1^* \left(S_{11} \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right)$$

And hence,

$$\frac{V_1^-}{V_1^*} = \Gamma_{in} = S_{11} \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$= \frac{S_{11} - S_{11}S_{22}\Gamma_L + S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$= \frac{S_{11} - \Gamma_L \left(S_{11}S_{22} - S_{12}S_{21} \right)}{1 - S_{22}\Gamma_L}$$

$$= \frac{S_{11} - \Gamma_L \Delta}{1 - S_{11}\Gamma_L} \tag{2.7}$$

Where Δ is the determination of the S-matrix = $S_{11}S_{22} - S_{12}S_{21}$

Similarly, it can be found that,

$$\Gamma_{out} = \frac{S_{22} - \Gamma_s \Delta}{1 - S_{11}\Gamma_s} \tag{2.8}$$

The input power to the amplifier is given by

$$P_{in} = \frac{|V_s|^2}{8R_s} M_s = \frac{|V_s|^2 R_{in}}{2|Z_S + Z_m|^2}$$

Where $|V_s|^2 / 8R_s$ is the available power from the source.

The source mismatch Ms is expressed in terms of Γ_s and Γ_{in}. From this, now the source impedance

$$Z_s = \frac{1 + \Gamma_s}{1 - \Gamma_s}$$

and $2R_s = Z_s + Z_s^* = \frac{1 + \Gamma_S}{1 - \Gamma_s} + \frac{1 + \Gamma_s^*}{1 - \Gamma_s^*} = \frac{2\left(1 - |\Gamma_s|^2\right)}{|1 - \Gamma_s|^2}$

Since, $(1 - \Gamma_s)(1 - \Gamma_s^*) = |1 - \Gamma_s|^2$

Similarly, $2R_{in} = \dfrac{2\left(1-|\Gamma_{in}|^2\right)}{|1-\Gamma_{in}|^2}$

Hence, the source mismatch from equation (2.3) can be obtain as

$$M_s = \frac{4R_{in}R_s}{|Z_s + Z_{in}|^2}$$

$$= \frac{4\left(1-|\Gamma_{in}|^2\right)\left(1-|\Gamma_s|^2\right)}{|1-\Gamma_{in}|^2|1-\Gamma_s|^2\left|\dfrac{1+\Gamma_s}{1-\Gamma_s}+\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}\right|^2}$$

$$= \frac{4\left(1-|\Gamma_{in}|^2\right)\left(1-|\Gamma_s|^2\right)|1-\Gamma_s|^2|1-\Gamma_{in}|^2}{|1-\Gamma_{in}|^2|1-\Gamma_s|^2\left|(1+\Gamma_s)(1-\Gamma_{in})+(1-\Gamma_s)(1+\Gamma_{in})\right|^2}$$

$$= \frac{\left(1-|\Gamma_{in}|^2\right)}{|1-\Gamma_s\Gamma_{in}|^2} \tag{2.9a}$$

Similarly, at the output the mismatch

$$M_L = \frac{\left(1-|\Gamma_{out}|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_L\Gamma_{out}|^2} \tag{2.9b}$$

The load voltage, $V_L = V_2^- + V_2^+ = V_2^-(1+\Gamma_l)$

The load currents, $I_L = Y_C\left(V_2^- - V_2^*\right)$

So, the power delivered to the load is given by,

$$P_L = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}Y_c\left|V_2^-\right|^2(1-\Gamma_L)\left(1-\Gamma_L^*\right)$$

Now, the input power is,

$$P_{in} = \frac{1}{2}\left|V_1^*\right|^2\left(1-|\Gamma_{in}|^2\right)Y_c$$

The input power and the available power are related

$$P_{in} = MP_{ava} = M_s P_{ava}$$

The transducer gain is given by,

$$G_t = \frac{P_L}{P_{ava}} = \frac{P_L}{|V_s|^2 / 8R_s}$$

Therefore, transducer gain is,

$$G_t = \frac{P_L}{P_{in}} M_s$$

$$= \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_{in}|^2\right)\left(1-|\Gamma_s|^2\right)}{\left(1-|\Gamma_{in}|^2\right)|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2}$$

$$= \frac{\left(1-|\Gamma_L|^2\right). \left(1-|\Gamma_{in}|^2\right)\left(1-|\Gamma_s|^2\right)|S_{21}|^2}{\left(1-|\Gamma_{in}|^2\right)\,|1-\Gamma_s\Gamma_{in}|^2}$$

From equation (2.7),

$$\Gamma_{in} = \frac{S_{11}\Delta\Gamma_L}{1-S_{22}\Gamma_L}$$

Therefore,

$$|\Gamma_{in}|^2 = \frac{|S_{11}-\Delta\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Or, $1-|\Gamma_{in}|^2 = 1-\dfrac{|S_{11}-\Delta\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$

$$= \frac{|1-S_{11}-\Delta\Gamma_L|^2 - |S_{11}-\Delta\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Therefore, $G_p = \dfrac{P_L}{P_{in}} = \dfrac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{\left(1-|\Gamma_{in}|^2\right)|1-S_{22}\Gamma_L|^2}$ (2.11)

$$= \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{\dfrac{|1-S_{22}\Gamma_L|^2 - |S_{11}-\Delta\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} x |1-S_{22}\Gamma_L|^2}$$

$$= \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{|1-S_{22}\Gamma_L|^2 - |S_{11}-\Delta\Gamma_L|^2}$$

Now,

$$|1-S_{22}\Gamma_L|^2 = (1-S_{22}\Gamma_L)(1-S^*{}_{22}\ \Gamma_L^*)$$

and $|S_{22}-\Delta\Gamma_L|^2 = (S_{22}-\Delta\Gamma_L)(S^*{}_{22}-\Delta^*\ \Gamma_L^*)$

Thus, the denominator of the equation for G_p is,

$|1-S_{22}\Gamma_L|^2 - |S_{11}-\Delta\Gamma_L|^2$

$$= (1-S_{22}\Gamma_L)(1-S^*{}_{22}\ \Gamma_L^*) - (S \ \ -\Delta\Gamma_L)(S^* - \Delta^*\Gamma_L^*)$$

$$= 1 - S_{22}^*\Gamma_L^* - S_{22}\Gamma_L + S_{22}S_{22}{}^*\Gamma_L\Gamma_L^* - S_{11}S_{11}^* + S_{11}\Delta^*\Gamma_L^* + \Delta\Gamma_L S_{11}^* - \Delta\Delta^*\Gamma_L\Gamma_L^* \left\{\because \Gamma_L\Gamma_L^* = |\Gamma_L|^2\right\}$$

$$= 1 - |S_{11}|^2 + |\Gamma_L|^2\left(|S_{22}|^2 - |\Delta|^2\right) - \Gamma_L^*\left(S_{22}^* - S_{11}\Delta^*\right) - \Gamma_L\left(S_{22} - \Delta S_{11}^*\right)$$

=

$$1 - |S_{11}|^2 + |\Gamma_L|^2\left(|S_{22}|^2 + |\Delta|^2\right) - 2\,\mathrm{Re}\left\{\Gamma_L\left(S_{22} - \Delta S_{11}^*\right)\right\}$$

$$G_p = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{1-|S_{11}|_2 + |\Gamma_L|^2\left(|S_{22}|^2 - |\Delta|^2\right) - 2R_e\left\{\Gamma_L\left(S_{22}-\Delta S_{11}{}^*\right)\right\}} \quad (2.12)$$

Similarly, the denominator is,

$$|1 - S_{11}\Gamma_L|^2 |1 - \Gamma_{in}\Gamma_s|^2$$

$$= |1 - S_{22}\Gamma_L|^2 \left| 1 - \left(\frac{S_{11-\Delta\Gamma_S}}{1 - S_{22}\Gamma_L} \right)\Gamma_S \right|^2 \qquad \left[\because \Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \right]$$

$$|1 - S_{22}\Gamma_L|^2 \frac{|1 - S_{22}\Gamma_L - (S_{11} - \Delta\Gamma_L)\Gamma_S|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= |1 - S_{22}\Gamma_L - S_{11}\Gamma_S + \Delta\Gamma_L\Gamma_S|^2$$

$$= |1 - S_{22}\Gamma_L - S_{11}\Gamma_S + (S_{11}S_{22} - S_{12}S_{21})\Gamma_L\Gamma_S|^2 \qquad [\because \Delta = S_{11}S_{22} - S_{12}S_{21}]$$

$$= |1 - S_{22}\Gamma_L - S_{11}\Gamma_S + (S_{11}S_{22}\Gamma_S\Gamma_L - S_{12}S_{21}\Gamma_S\Gamma_L)|^2$$

$$= |(1 - S_{22}\Gamma_L)(1 - S_{11}\Gamma_S) - (S_{12}S_{21}\Gamma_S\Gamma_L)|^2$$

Hence, $G = \dfrac{\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_S|^2\right)|S_{21}|^2}{|(1 - S_{22}\Gamma_L)(1 - S_{11}\Gamma_S) - S_{12}S_{21}\Gamma_S\Gamma_L|}$

As mentioned in section (2.4) when the device is absolutely stable, conjugate impedance matching can be used, that is,

$Z_s = Z_{in}^*$, $Z_L = Z_{out}$

In this case $\Gamma_S = \Gamma^*_{in}$, $\Gamma_L = \Gamma^*_{out}$ and $M_S = M_L = 1$

when $M_S = 1$, then $G_P = G$

For Conjugate Impedance Matching,

$$\left[\Gamma_S^* = \Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \right]$$

And $$\left[\Gamma_L = \Gamma^*{}_{out} = \frac{S_{11}{}^* - \Delta^*\Gamma_L{}^*}{1 - S_{22}{}^*\Gamma_L{}^*}\right]$$

Therefore,

$$\Gamma_{in} = \frac{S_{11} - \Delta\left(\dfrac{S_{22}^* - \Delta^*\Gamma_S^*}{1 - S_{11}^*\Gamma_S^*}\right)}{1 - S_{22}\left(\dfrac{S_{22}^* - \Delta^*\Gamma_S^*}{1 - S_{11}^*\Gamma_S^*}\right)}$$

$$= \frac{S_{11} - S_{11}S_{11}^*\Gamma_S^* - \Delta S_{22}^* - \Delta\Delta^*\Gamma_S^*}{1 - S_{11}^*\Gamma_S^* - S_{22}S_{22}^* + S_{22}\Delta^*\Gamma_S^*}$$

$$or, \Gamma_s^* = \frac{S_{11} - |S_{11}|^2\Gamma_S^* - \Delta S_{22}^* - |\Delta|^2\Gamma_S^*}{1 - S_{11}^*\Gamma_S^* - |S_{22}|^2 + S_{22}\Delta^*\Gamma_S^*}$$

$$or, \Gamma_S^* - S_S^*\left(\Gamma_S^*\right)^2 - |S_{22}|^2\Gamma_S^* + S_{22}\Delta^*\left(\Gamma_S^*\right)^2 = S_{11} - |S_{11}|^2\Gamma_S^* - \Delta S_{22}^* + |\Delta|^2\Gamma_S^*$$

$$or, \left(\Gamma_S^*\right)^2\left(S_{22}\Delta^* - S_{11}^*\right) + \Gamma_S^*\left(1 - |S_{22}|^2 + |S_{11}|^2 - |\Delta|^2\right) + \Delta S_{22}^* - S_{11} = 0$$

Therefore,

$$\Gamma_s^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad where, a = s_{22}\Delta^* - s^*{}_{11}$$

$$= \frac{A_1}{2B_1} \pm \sqrt{A_1^2 - 4|B_1|}^{\,2} \qquad b = 1 - |S_{11}|^2 + |S_{11}|^2 - |\Delta|^2$$

$$c = \Delta S_{22}^* - S_{11}$$

So, the above equation becomes

$$\Gamma_S = \Gamma_{SM} = \frac{1}{2B_1\left(A_1 \pm 4|B_1|^2\right)} \tag{2.13a}$$

Similarly,

$$\Gamma_L = \Gamma_{LM} = \frac{1}{2B_2}\left(A_2 \pm \sqrt{A_2^2 - 4|B_2|^2}\right) \tag{2.3b}$$

Where

$$A_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$B_2 = S_{22} - S_{11}^*\Delta$$

For an absolutely stable device $A_1 > 0$ and $A > 0$. The minus sign when $A_1 > 0$ and the plus sign is used when $A_l < 0$.

Thus, the negative sign is the appropriate one to use in equation (2.13).

When

$$A_2^2 < 4|B_2|^2$$

Then

$$\Gamma_{LM} = \frac{A_2 \pm j\sqrt{4|B_2|^2 - A_2^2}}{2B_2}$$

$$= \frac{A_2}{2B_2} \pm \frac{j\sqrt{4|B_2|^2} - A_2^2}{2B_2}$$

$$= \sqrt{\left(\frac{A_2}{2B_2}\right)2 + \left(\frac{\sqrt{4|B\ |^2 - A_2^2}}{2B_2}\right)^2}$$

$$= \sqrt{\frac{A_2^2}{4|B_2^2|} + \frac{4|B_2|^2 - A_2^2}{4|B_2^2|}}$$

$$= \sqrt{\frac{A_2^2 + 4|B_2|^2 - A_2^2}{4|B_2^2|}}$$

$$= 1$$

Therefore,

$$G_P = \frac{\left(1 - |\ \Gamma_L\ |^2\right)|S_{21}|^2}{\left(1 - |\Gamma_{in}|^2\right)|1 - S_{22}\Gamma_L|^2}$$

$$= 0 \qquad\qquad [since, |\Gamma_L| = 1]$$

So, for an absolutely stable device one should have,

$$A_2^2 > 4|B_2|^2$$

$$or,\ |A_2| > 2|B_2|$$

A device is absolutely stable if the stability parameter (K) is greater than 1, that is, stability parameter,

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1 \qquad (2.14)$$

In terms of parameter K, the power gain G_p can be derived out as follows

By direct expansion,

$$A_2^2 - 4|B_2|^2 \;= 4|S_{12}S_{21}|^2 - (K^2 - 1)$$

Now $\quad A_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$

Or, $A_2 = |S_{22}|^2 + |\Delta|^2 = 1 - |S_{11}|^2$

Now it can be proved that,

$$1 - |S_{11}|^2 = A_2 - |S_{22}|^2 + |\Delta|^2 = 2K|S_{12}S_{21}| - (\|\Delta\|^2 - |S_{22}|^2)$$

Proof:

Now $\quad 2K|S_{12}S_{21}| - (|\Delta|^2 - |S_{22}|^2)$

$$= 2\left[\frac{(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2)X|S_{12}S_{21}|}{2|S_{12}S_{21}|}\right] - |\Delta\;|^2 + |S_{22}|^2$$

$= 1 - |S_{11}|^2 \qquad$ Putting value of K from equation (2.14)

Therefore,

$$1 - |S_{11}|^2 = K|S_{12}S_{21}| + A_2/2$$

and $\quad |\Delta|^2 - |S_{11}|^2 = K|S_{12}S_{21}| - A_2/2$

From equation (2.13b)

$$2\Gamma_{LM} B_2 = A_2 \pm \sqrt{A_2^2 - 4|B_2|^2}$$

But $\quad B_2 = S_{22} - \Delta S_{11}^*$

Therefore,

$$2\Gamma_{LM}\left(S_{22}-\Delta S_{11}^{*}\right)=A_2 \pm \sqrt{A_2^2-4|B_2|^2}, \quad \textit{which is a real quality,}$$

Therefore,

$$2\,\mathrm{Re}\,\Gamma_{LM}\left(S_{22}-\Delta S_{11}^{*}\right)=2\Gamma_{LM}\left(S_{22}-\Delta S_{11}^{*}\right)$$

Since, $\Gamma_L = \Gamma_{LM,}$ when matched

Therefore,

$$2\,\mathrm{Re}\,\Gamma_L\left(S_{22}-\Delta S_{11}^{*}\right)=2\Gamma_L\left(S_{22}-\Delta S_{11}^{*}\right)=A_2 \pm \sqrt{A_2^2-4|B_2|^2}$$

Since from equation (2.6a)

$$A_2^2-4|B_2|^2 = 4|S_{12}S_{21}|^2\left(K^2-1\right)$$

Therefore,

$$2\Gamma_L\left(S_{22}-\Delta S_{11}^{*}\right)=A_2 \pm 2|S_{12}S_{21}|\left(K^2-1\right)$$

Using these expressions, the denominator in Equation. (2.12), which is the equation for p, we get,

$$1-|S_{11}|^2+|\Gamma_L|^2\left(|S_{22}|^2-|\Delta|^2\right)-2\,\mathrm{Re}\,\Gamma_L\left(S_{22}-\Delta S_{11}^{*}\right)$$

$$=K|S_{12}S_{21}|+{A_2}/{2}+|\Gamma_L|^2\left({A_2}/{2}-K|S_{12}S_{21}|\right)-A_2 \pm 2|S_{12}S_{21}|\sqrt{K^2-1}$$

$$=-{A_2}/{2}\left(1-\Gamma_L^2\right)+\left(1-\Gamma_L^2\right)\left(K|S_{12}S_{21}|\right)\pm 2|S_{12}S_{21}|\sqrt{K^2-1}$$

$$=\left(1-|\Gamma_L|^2\right)\left[\frac{\pm 2|S_{12}S_{21}|\sqrt{K^2-1}}{\left(1-|\Gamma_L|^2\right)}-\left(\frac{A_2}{2}-K|S_{12}S_{21}|\right)\right]$$

Now the step is used to get the solution Γ_{LM} for Γ_L as

$$\Gamma_{LM} = \frac{A_2 \pm \sqrt{A_2^2 - 4|B_2|}^2}{2B_2}$$

$$\Gamma_{LM}^2 = \frac{1}{4|B_2|^2}\left\{A_2^2 + 4|S_{12}S_{21}|^2\left(K^2 - 1\right) \pm 2A_2\sqrt{4|S_{12}S_{21}|^2\left(K^2 - 1\right)}\ \right\}$$

$$or, 1 - \Gamma_L^2 = \frac{4|B_2|^2 - A_2^2 - 4|S_{12}S_{21}|^2\left(K^2 - 1\right) \pm 2A_2\sqrt{4|S_{12}S_{21}|^2\left(K^2 - 1\right)}}{4|B_2|^2}$$

$$= \frac{-4|S_{12}S_{21}|^2\left(K^2 - 1\right) - 4|S_{12}S_{21}|^2\left(K^2 - 1\right) \pm 2A_2\sqrt{4|S_{12}S_{21}|^2}\left(K^2 - 1\right)}{4|B_2|^2}$$

$$= \frac{-8|S_{12}S_{21}|^2\left(K^2 - 1\right) \pm 2A_2\sqrt{4|S_{12}S_{21}|^2\left(k^2 - 1\right)}}{4|B_2|^2}$$

$$= \frac{-2|S_{12}S_{21}|^2\left(K^2 - 1\right) \pm A_2\sqrt{|S_{12}S_{21}|^2\left(k^2 - 1\right)}}{|B_2|^2}$$

$$= \frac{2|S_{12}S_{21}|\sqrt{k^2 - 1}\left\{-|S_{12}S_{21}|\sqrt{K^2 - 1 \pm A_2}\right\}}{|B_2|^2}$$

$$= \frac{2|S_{12}S_{21}|\sqrt{k^2 - 1}\left\{\pm A_2 - |S_{12}S_{21}|\sqrt{K^2 - 1}\right\}}{|B_2|^2}$$

After substituting this we, get,

$$G_P = \frac{|S_{21}|^2}{|S_{12}S_{21}\ |\left(K \pm \sqrt{K^2 - 1}\right)}$$

$$= \frac{|S_{21}|^2\left(K \pm \sqrt{K^2-1}\right)}{|S_{12}S_{21}|\left(K^2 \pm K^2-1\right)}$$

$$= \left|\frac{S_{21}}{S_{12}}\right|\left(K \pm \sqrt{K^2-1}\right)$$

Since G_p should become zero when K=1

Therefore, $$G_P = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2-1}\right) \qquad (2.15)$$

Which corresponds to the gain obtained with passive load?

2.4 AMPLIFIER STABILITY CRITERIA

When the stability parameter K<1, the transistor is potentially unstable. A stable amplifier can still be designed but only for some restricted values of source and load impedances. Also, in this condition it is not possible to use conjugate impedance matching at both the input and the output ports as mentioned before.

In this section expressions for allowed terminating impedances are derived in terms of input and output reflection coefficient stability circles. These stability circles can be plotted on a smith chart and will show what values of source and load impedances can be used in order to achieve a stable, that is, non-oscillating amplifier. The smith chart is an indispensable aid in the visualization of the different constraints that the engineer must take into account in the design of a microwave amplifier. The smith chart is an indispensable aid in the visualization of the different constraints that the engineer must take into account in the design of a microwave amplifier.

The conditions for amplifier stability are established by requiring that the reflected power from the amplifier ports be smaller than the incident power. This means that the reflection coefficients looking into the amplifier ports must have a magnitude <1 for all passive source and load impedances.

If |reflection coefficient| >1, then the amplifier input or output impedance would have a negative real part, for example,

If Z_{in} = -R + jX, then,

$$|\Gamma_{in}| = \left|\frac{-R + jX - Z_c}{-R + jX + Z_c}\right|$$

$$= \sqrt{\frac{(R + Z_c)^2 + X^2}{(Z_c - R)^2 + X^2}} \rangle 1$$

$$I = \frac{V_s}{R_s - R_{in} + j(X_{in} + X_s)}$$

If $R_s = R_{in}$ and $X_{in} + X_s = 0$ (which occurs at some frequency) then I becomes infinite. Now setting Vs = 0 and the thermal noise in the input can produce self sustained oscillations at the frequency where the total loop intendance at the input equals 0. Oscillations at any frequency generally make the amplifier unusable.

Thus, the condition for stability is [1]:

$$|\Gamma_{in}| = \left|\frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1 \qquad \text{for all} |\Gamma_{11}| < 1 \tag{2.16a}$$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s}\right| < 1 \qquad \text{for all} |\Gamma_s| < 1 \tag{2.16b}$$

When these conditions are satisfied conjugate impedance matching can be used.

If $Z_L = Z_C$, then $\Gamma_L = 0$ and $|\Gamma_{in}|<1$ only if $|S_{11}|<1$.

Similarly, if $Z_s = Z_C$, $|\Gamma_{out}|<1$ only if $|S_{22}|<1$.

Therefore, the necessary condition for absolute stability is

$|S_{11}|<1$ and $|S_{22}|<1$

The values of Γ_L that result in $|\Gamma_{in}| <1$ are called stable values and the corresponding region in the Smith Chart is the stable region. The boundary between stable values of Γ_L and unstable values in the Γ_L place is a circle corresponding to the mapping of the circle $|\Gamma_L| = 1$ in the Γ_{in} plane.

$$\Gamma_{11} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{\Delta\Gamma_L - S_{11}}{S_{22}\Gamma_L - 1}$$

Thus, the center of the mapped circle is at Δ = A,-S_{11}=B, S_{22}=C,-1=D, ρ=1

$$\Gamma_{LC} = \frac{S_{22}(-1) - \Delta^*(-S_{11})}{|\Delta|^2 - |S_{22}|^2}$$

$$= \frac{S_{11}\Delta^* - S_{22}^{\ *}}{|\Delta|^2 - |S_{22}|^2} \qquad (2.17a)$$

and the radius of the circle is R_{LC}

$$= \frac{\rho|AD - BC|}{\left||A|^2 - \rho^2|C|^2\right|}$$

$$= \frac{|\Delta(-1) - (-S_{11})S_{22}|}{\left||\Delta|^2 - |S_{22}|^2\right|}$$

$$= \frac{|S_{11}S_{22} - \Delta|}{\left||\Delta|^2 - |S_{22}|^2\right|}$$

$$= \frac{|S_{11}S_{22} - S_{11}S_{22} + S_{12}S_{21}|}{\left||\Delta|^2 - |S_{22}|^2\right|} \qquad (\because \Delta = \mathrm{S}_{11}S_{22} - S_{12}S_{21})$$

$$= \frac{|S_{12}S_{11}|}{\left||\Delta|^2 - |S_{22}|^2\right|}$$

The load stability circle may or may not include the origin $\Gamma_L = 0$ as shown in the Figure 2.3

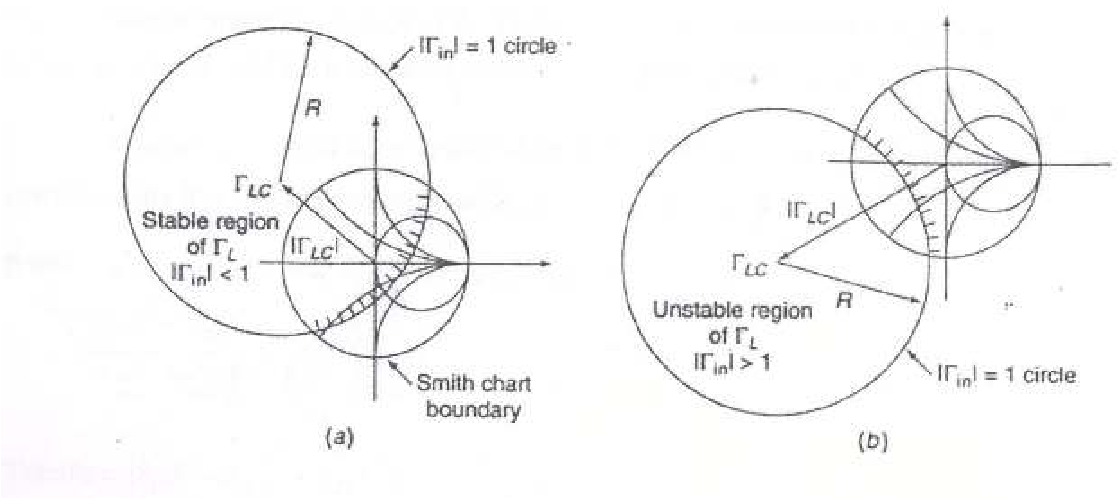


Figure2.3: Load-Stability Circle

When $\Gamma_L = 0$, $\Gamma_{in} = S_{11}$ and $|S_{11}|<1$ for a stable circuit. Hence, if the stability circle encloses the origin, then all values of Γ_L are stable ones. If $\Gamma_L = 0$ lies outside the circle, then all values of Γ_L outside the circle are stable ones. The origin is included only if the radius $R_{LC}>|\Gamma_{LC}|$ If this situation prevails then, $R_{LC}>|\Gamma_{LC}|$, so that all values of $|\Gamma_{LC}|\leq 1$ lie inside the circle and represent stable values for all possible passive load terminations.$|\Gamma_{LC}|>1+R_{LC}$, so that the entire stability circle $|\Gamma_{LC}| =1$ maps into a circle outside the Smith chart boundary $|\Gamma_{LC}|=1$. Then all values of $|\Gamma_{LC}|<1$ will be stable .We need to find out expressions by using scattering matrix parameters of the device that tells whether the device is absolutely stable or not. If the amplifier is stable for all passive source and load impedances, then the device is absolutely stable. If it is stable only for a limited set of source and load impedances, then the device is conditionally stable.

2.4.1 Expression for stability: Case study

Let us now consider two cases and they both lead to the same statement or criterion for stability.

Case 1

Let origin (Γ_L=0) lies outside the circles of Γ_L values that make $|\Gamma_{in}|$=1, that is, center or origin lies outside of the circle $|\Gamma_{in}|$=1. It is mentioned before that all values of Γ_L outside the load stability circle $|\Gamma_{in}|$=1 are the stable values [1]:

All values of ZL will be acceptable if the $|\Gamma_{in}|$=1 circle lies outside the Smith chart boundary $|\Gamma_L|$=1. This requires that $|\Gamma_{LC}|>1+R_{LC}$, that is, $|\Gamma_{LC}|^2>R_{LC}^2$

Putting the values of Γ_{Lc} and R_{Lc} from equation (2.17),

$$\frac{\left|S_{11}\Delta^* - S_{22}^*\right|}{\left(\left|\Delta\right|^2 - \left|S_{11}\right|^2\right)^2} > \frac{\left|S_{12}S_{21}\right|^2}{\left(\left|\Delta\right|^2 - \left|S_{11}\right|^2\right)}$$

Therefore,

$$\left|S_{11}\Delta^* - S_{21}\right|^2$$

By the direct expansion it may be seen that

$$\left|S_{11}\Delta^* - S_{22}^*\right| = \left|S_{12}S_{21}\right|^2 + \left(1 - \left|S_{11}\right|^2\right)\left(1 - \left|S_{22}\right|^2 - \left|\Delta\right|^2\right) \qquad (2.18)$$

Putting this in the above

$$\left(1 - \left|S_{11}\right|^2\right)\left(\left|S_{22}\right|^2 - \left|\Delta\right|^2\right) > 0$$

Since $\left|S_1\right| < 1$

Therefore,

, $\left|S_1\right| < \left|\Delta\right|$

The stability condition can also be written as

$$1 + R_{LC} < \left|\Gamma_{LC}\right|$$
$$= \left(1 + R_{LC}\right)^2 < \left|\Gamma_{LC}\right|^2$$

$$= \frac{\left|S_{11}\Delta^* - S_{22}^*\right|}{\left(\left|\Delta\right|^2 - \left|S_{22}\right|^2\right)^2} > \left(1 + R_{LC}\right)^2$$

$$= \left|S_{11}\Delta^* - S_{22}^*\right| > \left(1 + R_{LC}\right)^2 \left(\left|\Delta\right|^2 - \left|S_{22}\right|^2\right)^2$$

By putting equation (2.17b) we get

$$1 + R_{LC} = 1 + \frac{\left|S_{12}S_{21}\right|}{\left|\left|\Delta\right|^2 - \left|S_{22}\right|^2\right|}$$

$$= \frac{\left|S_{22}\right|^2 - \left|\Delta\right|^2 + \left|S_{12}S_{21}\right|}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}$$

Putting value of (1+R_{LC})

$$\left|S_{11}\Delta^* - S_{22}^*\right|^2 > \frac{\left(\left|S_{22}\right|^2 - \left|\Delta\right|^2 + \left|S_{12}S_{21}\right|\right)^2}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} X \left(\left|\Delta\right|^2 - \left|S_{22}\right|^2\right)^2$$

Therefore, $\left|S_{11}\Delta^* - S_{22}^*\right|^2 > \left(\left|S_{22}\right|^2 - \left|\Delta\right|^2 + \left|S_{12}S_{21}\right|\ \right)^2$

$$+\left|S_{11}\Delta^* - S_{22}^*\right|^2 > \frac{\left(\left|S_{22}\right|^2 - \left|\Delta\right|^2 + \left|S_{12}S_{21}\right|\right)^2}{} X \left(\left|\Delta\right|^2 - \left|S_{22}\right|^2\right)^2$$

Or, $\left(\left|S_{22}\right|^2 - \left|\Delta\right|^2\right) + \left(2\left|S_{12}S_{21}\right|\right) < 1 - \left|S_{11}\right|^2$

The stability parameter,

$$K = \frac{1 - \left|S_{11}\right|^2 - \left|S_{22}\right|^2 + \left|\Delta\right|^2}{2\left|S_{12}S_{21}\right|} > 1 \qquad (2.19a)$$

$$\left|S_{11}\right| < 1 \qquad (2.19b)$$

Case 2

It may happen that the circle of Γ_L values giving $|\Gamma_L|=1$ encloses the origin ($\Gamma_L = 0$, Γ_{in} = S11). In this case all Γ_L values inside the circle are stable ones. For absolutely stable devices the circle must than be large enough to enclose the entire smith chart $|\Gamma_L| \leq 1$, so that any passive load impedance Z_L can be used. This requires that the radius R_{LC} should be greater than 1.

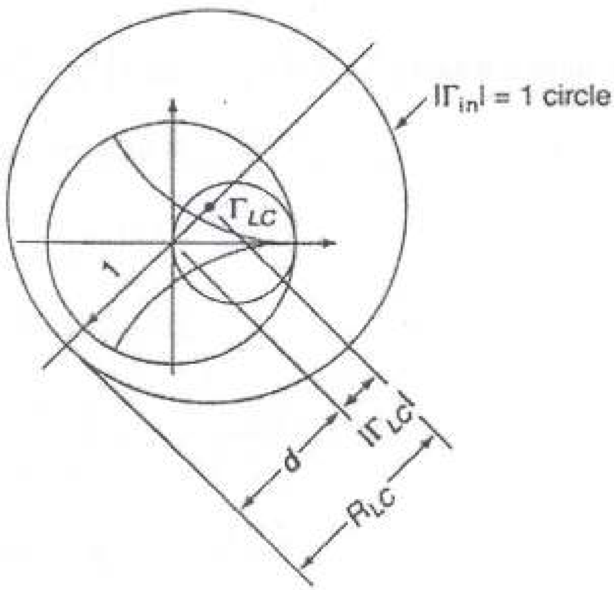


Fig. 2.4 Stability circle encloses the origin

From figure (2.4), it is clearly seen that,

$d>1$

Or, $R_{LC} - |\Gamma_{Lc}| > 1$

Or, $|\Gamma_{Lc}| < R_{LC} - 1$

This occurs only if

$|\Gamma_{Lc}| < R_{LC}$

From equation 21),

$|S_{22}| < |\Delta|$

Therefore, by using equations (2.17 a), (2.17b) and (2.18), the stability conditions may be obtained in the form.

$$\left[\left(|\Delta|^2 - |S_{22}|^2\right) - |S_{12}S_{21}|\right]^2 > |S_{12}S_{21}|^2 + \left(1 - |S_{11}|^2\right)\left(|S_{22}|^2 - |\Delta|^2\right)$$

or,

$$2|S_{12}S_{21}|\ \left(|\Delta|^2 - |S_{22}|^2\right) > |S_{12}S_{21}|^2 - \left(1 - |S_{11}|^2\right)\left(|\Delta|^2 - |S_{22}|^2\right)$$

Or, $\left(|\Delta|^2 - |S_{22}|^2\right) - 2|S_{12}S_{21}|^2 > \left(1 - |S_{11}|^2\right)$

Or, $\left(|S_{22}|^2 - |\Delta|^2\right) + 2|S_{12}S_{21}| < 1 - |S_{11}|^2;$

Which is same as that for case –1 before?

For absolute stability R $_{LC}$ >1

Or, $|S_{12}S_{21}| > |\Delta|^2 - |S_{22}|^2;$

From equation (2.19),

$$= \frac{1 - |S_{11}|^2}{|S_{12}S_{21}|} + \frac{|\Delta|^2 - |S_{22}|^2}{|S_{12}S_{21}|} = 2K$$

Now, since $|S_{12}S_{21}| < |\Delta|^2 - |S_{22}|^2;$

Therefore, $\dfrac{|\Delta|^2 - |S_{22}|^2}{|S_{12}S_{21}|} < 1$

Let $\dfrac{|\Delta|^2 - |S_{22}|^2}{|S_{12}S_{21}|} = 1 - \sigma$

Where σ = a positive quantity

Therefore, $\dfrac{1 - |S_{22}|^2}{|S_{12}S_{21}|} = 2K - 1 + \sigma > 1$ (since K>1)

Hence, $|S_{12}S_{21}| < 1 - |S_{11}|^2;$

The source stability circle is the circle of source reflection – coefficient Γs values that make $|\Gamma_{out}|$ =1. By direct analogy with the derivation of Equation (2.17) it may be found out that the center for the source stability circle, in the Γ_s plane is given by,

$$\Gamma_{sc} = \frac{S_{22}\Delta^* - S^*_{11}}{|\Delta|^2 - |S_{11}|^2} \tag{2.20a}$$

And its radius is,

$$R_{sc} = \frac{|S_{12}S_{21}|}{\left|\,|\Delta|^2 - |S_{11}|^2\,\right|} \tag{2.20b}$$

These equations are same as equation (2.17) with S_{11} and S_{22} interchanged. Since K is symmetrical in the variables S_{11} and S_{22} , it can be inferred that the same condition >1 for absolute stability is obtained from the requirement $|\Gamma_{out}| < 1$, along with the conditions,

$$|S_{22}| < 1 and |S_{12}S_{21}| < 1 - |S_{22}|^2$$

Thus the necessary and sufficient conditions for absolute stability are

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} < 1 \tag{2.21a}$$

$$|S_{11}| < 1 \tag{2.21b}$$

$$|S_{22}| < 1 \tag{2.21c}$$

$$|S_{12}S_{21}| < 1 - |S_{11}|^2 \tag{2.21d}$$

For most devices $|\Delta| < 1$, so equation (2.21a) to (2.21c) is sufficient to guarantee absolute stability.

Constant Power-Gain Circles

The normalized power gain (g_p) may be defined as [2]:

$$g_p = \frac{G_p}{|S_{21}|^2} \tag{2.22}$$

From equation (2.12), normalized gain may be defined as

$$g_p = \frac{1 - \Gamma_L\Gamma_L^*}{\left[1 - |S_{11}|^2 + \Gamma_L\Gamma_L^*\left(|S_{22}|^2 - |\Delta|\right) - \Gamma_L\left(S_{22} - \Delta S_{11}^*\right) - \Gamma_L^*\left(S_{22}^* - \Delta^* S_{11}\right)\right]} \tag{2.23}$$

Or,

$$g_p - g_p|S_{11}|^2 + g_p\Gamma_L\Gamma_L^*\left(|S_{22}|^2 - |\Delta|^2\right) - \Gamma_L g_p(S_{22} - \Delta S_{11}^*) - \Gamma_L^* g_p(S_{22}^* - \Delta^* S_{11}) = 1 - \Gamma_L\Gamma_L^*$$

Or,

$$\Gamma_L\Gamma_L^*\{1+g_p\left(|S_{22}|^2-|\Delta|^2\right)\}-[\ )\Gamma_L g_p(S_{22}-\Delta S_{11}^*)+\Gamma_L^* g_p(S_{22}^*-\Delta^* S_{11})-g_p\left(1-|S_{11}|^2\right)+1\ |=0$$

$$\text{Or, } \Gamma_L\Gamma_L^*-\left[\frac{\Gamma_L g_p\left(S_{22}-\Delta S^*{}_{11}\right)+\Gamma_L^* g_p\left(S_{22}^*-\Delta^* S_{11}\right)-g_p\left(1-|S_{11}|^2\right)+1}{g_p\left(|S_{22}|^2-|\Delta|^2\right)+1}\right]=0$$

$$\text{By doing } \left[\frac{g_p X|S_{21}|^2}{\left(1-|\Gamma_L|^2\right)\Gamma_L\Gamma_L^*}\right]$$

Comparing with equation (1.1) it may be seen that it describes a circle in the Γ_L Plane. The center of the circle is given by,

$$\Gamma_{Lg}=\frac{(S^*{}_{22}-\Delta^* S_{11})g_P}{\left(|S_{22}|^2-|\Delta|^2\right)g_p+1} \qquad (2.24a)$$

And the radius is given by

$$R_{Lg}=\frac{(1-2Kg_p|S_{12}S_{21}\ |\ +g_p^2|S_{12}S_{21}|^2)^{½}}{\left(|S_{22}|^2-|\Delta|^2\right)g_p+1} \qquad (2.24b)$$

When $g_p = 0$, then $\Gamma_{Lg} = 0$, $\Gamma_{Lg} = 1$, which is the boundary of the smith chart

It is common practice to plot the normalized constant power - gain circles that correspond to gains 1dB, 2 dB. 3dB etc. less than the maximum normalized power gain for an absolutely stable device is given by $g_p=\frac{G_p}{|S_{21}|^2}=\frac{1K-\sqrt{K^2-1}}{|S_{12}S_{21}|}$

By plotting the constant power-gain circles and the load stability circle, one can easily determine the value Γ_L that will give the largest power gain and yet result in a stable amplifier design. Those value of Γ_L in a stable region and lying on a given g_p = constant circle will give a power gain

2.5 BASIC NOISE THEORY

Due to thermal agitation, the electrons in a resistor have an inherent random motion, which results in a random voltage appearing across the resistor terminals. This random voltage is called noise. There is no analytical way to describe the exact voltage waveform, so one must be content with a description of certain average characterization of the noise. Thus it is possible to model a noisy resistor as a noise free resistor in series with a noise voltage generator $e_n(t)$ or in shunt with a noise current source $i_n(t)$ as shown in the figure below.

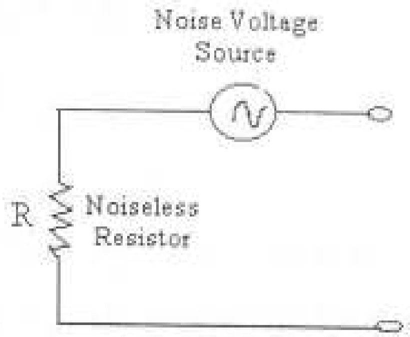


Fig.2.5a Thevenin Equivalent Circuit, which uses a Noise voltage generator

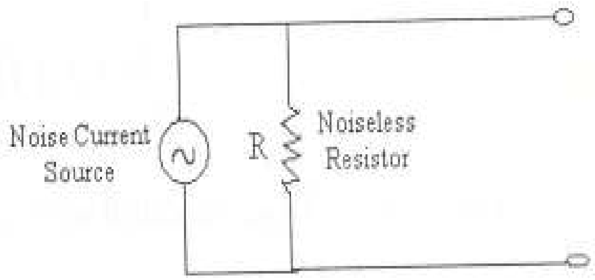


Fig.2.5b Equivalent Circuit for a Noisy resistor in which a noise current source is used.

Thermal noise is generally regarded as a stationary ergodic noise process, which is a random process for which ensemble averages can be replaced by time average. Thus, in this discussion about noise theory time averages is used. Since, the objective is only to obtain the basic results needed to derive the equations required for low-noise amplifier design.

The time average value of the noise voltage is given by,

$$\langle e_n(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} e_n(t)dt = 0 \tag{2.25}$$

The correlation function for the noise voltage is the average value of the product of the noise voltage at time t and that at a later (t+ τ), thus is,

$$C(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} e_n(t) e_n(t+\tau) dt$$
$$= \langle e_n(t)_{e_n}(t+\tau) \rangle \qquad (2.26)$$

Where C (τ) = correlation function.

If τ = 0, the average power $\langle e_n^2 \rangle$ associated with the noise (the noise voltage is through of as being applied to a 1 Ω resistor so that the dimensions are those of power). The average power in noise is distribution over a broad band of frequencies because noise voltage waveforms contain a broad spectrum of frequencies.

The power spectral density of noise is,

$$S_n(W) = \int_{-T}^{T} C(\tau) e^{-jw\tau} dt \qquad (2.27a)$$

The inverse transform relationship is,

$$C(\tau) = \int_{-T}^{T} S_n(W) e^{jw\tau} \frac{dw}{2\pi} \qquad (2.27b)$$

So, the noise power in the frequency (Δf) is {Sn (W) Δf}.

Since, the power spectral density of thermal noise is constant up to 1000 GHZ, so, it can be assumed that the density is constant at microwave frequencies and below.

The power spectral density is an even function of w, so only positive values of w need to be considered. For thermal noise in a resistor, the power spectral density for the noise voltage is given by the Nyqueist's formula as

S_e (W) = 4KTR w≥0 (2.28)

Where K= Boltzmann's constant = 1.38 x 10 $^{-2}$3 j/K

T = Absolute temperature of the resistor 'R'

R= resistance

Thus, the amount of noise power, P_n in a frequency range (Δ f) is given by

P_n = 4KTRΔf (2.29)

For the equivalent current source model shown in figure (1b), the average power, if the current I n (t) flows in a 1 Ω resistor is given by $\langle\ i_n^2(t)\rangle$ and has a power spectral density given by,

$$S,(W) = \frac{4WT}{R} \qquad w\geq 0 \qquad (2.30)$$

Filtered Noise

Consider a sinusoidal voltage generator with a complex r.m.s. voltage, V that is connected in series with a source impedance Z_s and a two-port network as shown in figure below, where Z_{in} is the input impedance of the network.

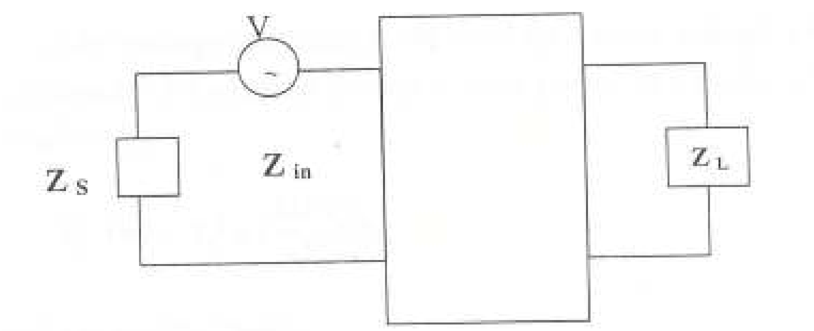


Fig.2.6a A Two-port Network connected to a Voltage Source.

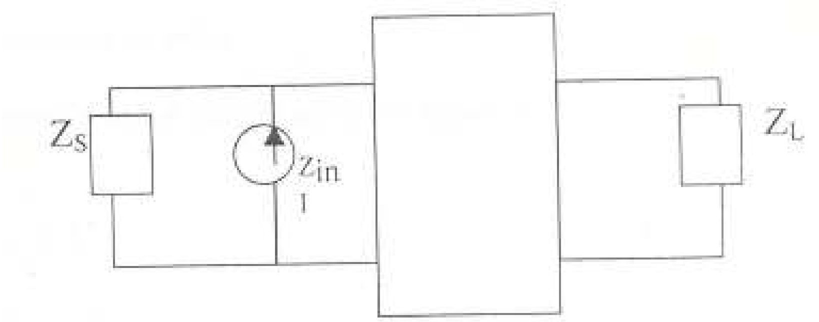


Fig.2.6b A Two-port Network connected to a Current Source.

The input current is $\frac{V}{(Z_S + Z_{in})}$

The input power produced by V is given by,

$$P_{in} = \left\| \frac{V}{Z_S + Z_{in}} \right\|^2 R_{in} = \frac{|V^2|}{4R_S} \frac{4R_S R_{in}}{|Z_S + Z_{in}|^2} = \frac{|V|^2}{4R_S} M \qquad (2.31)$$

Where,

Available power = $\frac{|V^2|}{4R_S}$

Impedance mismatch factor = M

Power transfer function and is a function of w as Z_s and Z_{in} are functions of

$$w = \frac{M}{4R_s}$$

If the voltage generator is replaced by a noise voltage source e n (t) with power spectral density Se(w), then the input noise power in a frequency band (Δ f) centered on w is given by

$$P_{n,e}(W) = S_e(W) \frac{M(w)}{4R_s} \Delta f \qquad (2.32)$$

The total input noise power is,

$$P_{NT}(\tau) = \int_0^\infty S_e(W) \frac{M(w)}{4R_e} \frac{dw}{2\pi} \qquad (2.33)$$

Now consider the circuit below,

The input current from the current source is equal to,

$$\frac{Z_s I}{(Z_S + Z_{in})}$$

And the input power is,

$$\begin{aligned} P_{in,2} &= \left\| \frac{IZ_S}{Z_S + Z_{in}} \right\|^2 R_{in} = \frac{|I|^2 |Z_S|^2}{4R_S} \frac{4R_S R_{in}}{|Z_S + Z_{in}|^2} \\ &= \frac{|I|^2}{4G_S} m \end{aligned} \qquad (2.34)$$

Where $G_S = \frac{R_S}{|Z_S|^2}$

If the sinusoidal current is replaced by a noise current source $i_n(t)$ with power spectral density $S_i(w)$, then the input noise power in a frequency range (Δf) is given by

$$P_{n,i}(W) = S_i(W)\frac{M(w)}{4G_s}\Delta f \qquad (2.35)$$

When both sources V and I are acting, the input current will be

$$I_{in} = \frac{V}{Z_S + Z_{in}} + \frac{IZ_S}{Z_S + Z_{in}}$$

$$= \frac{V + IZ_S}{Z_S + Z_{in}}$$

And the input power is given by,

$$P_{in,3} = \left(\frac{V + IZ_S}{Z_S + Z_{in}}\right)^2 R_{in}$$

$$= \frac{|V + IZ_S|^2}{|Z_S + Z_{in}|^2} R_{in}$$

$$= \frac{(V + IZ_S)(V^* + I^* Z^*{}_S)}{|Z_S + Z_{in}|^2} R_{in}$$

$$= \frac{\left(|V|^2 + VI^* Z_S^* + IZ_S V^* + I^2 Z_S^2\right)}{|Z_S + Z_{in}|^2} R_{in}$$

Now multiplying (4Rs) in the numerator and the denominator we get,

$$= \frac{4R_S |V|^2}{|Z_S + Z_{in}|^2}\frac{R_{in}}{4R_S} + \frac{|I|^2 |Z_S|^2 4R_S R_{in}}{4R_S |Z_S + Z_{in}|^2} + 2R_E\left[\frac{VI^* Z^*{}_S R_{in}}{|Z_S + Z_{in}|^2}\right]$$

$$= \frac{|V|^2}{4R_S} M + \frac{|I|^2 M}{4G} + 2R_E \frac{VI^* Z_S^* R_{in}}{|Z_S + Z_{in}|_2} \qquad (2.36)$$

Because of interruption between the two sources, the input power is not only the sum of power from each source acting independently.

When 2 noise source $e_n(t)$ and $i_n(t)$ are acting simultaneously, there is no input power caused by the two sources if they are uncorrected. When uncorrected, then total power input = power input by the first source + power input by the second source.

When there is a degree of correlation between the two sources, then there will be some input noise power due to the source interaction.

The cross-correlation between the current source $i_n(t)$ and voltage source $e_n(t)$ is given by,

$$C_X(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} i_n(t) e_n(t+\tau) dt \qquad (2.37a)$$

The Fourier transform of $C_x(\tau)$ gives the cross-power spectral $S_x(w)$, that is,

$$S_X(W) = S_{xr}(W) + jS_{xi}(w) = \int_{-\infty}^{\infty} C_X(\tau) e^{-jw\tau} d\tau \qquad (2.37b)$$

If w is replaced by (-w), it may be seen that $S_x(-w) = S^*_x(w)$, since $C_x(\tau)$ is real. From this result it may be found that $S_{xr}(w)$ is an even function of w and $S_{xi}(w)$ is an odd function of w. for input noise power calculations, $|V|^2$ is replaced by the noise voltage source power spectral density $S_e(w)$ and $|I|^2$ by the power spectral density $S_x(w)$ in equation (2.36). Thus, for partially correlated noise sources, the total input noise in a frequency band (Δf) is given by,

$$P_n = \Delta f \left\{ S_E(w)\frac{M}{4R_S} + S_i(w)\frac{M}{4G_S} + \frac{4[S_{xr}(w)R_S + S_{xi}(w)X_S]}{|Z_S + S_{in}|^2} R_{in} \right\} \qquad (2.38)$$

The extra factor '2' in the last term is due to the combination of he contribution from negative values of w with those from positive value off w using the fact that and $S_{ix}X_s$ are even functions of W.

From equation (2.38), the power spectral density of the noise power delivered to 'Zl' is found as,

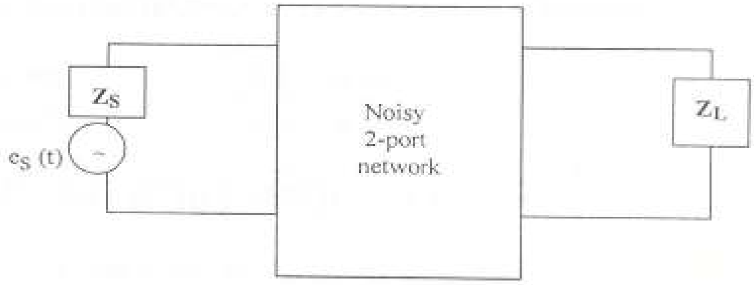
$$S(w) = \left\{ \frac{M}{4R_S} S_e(w) + \frac{M}{4G_S} + S_i(w) + \left[S_{xr}(w) + \frac{X_S}{R_S} S_{xi}(w) \right] M \right\} G_p(w) \qquad (2.39)$$

After multiplying the power gain G_p of the 2-port network, the total output noise power delivered to Z_L is

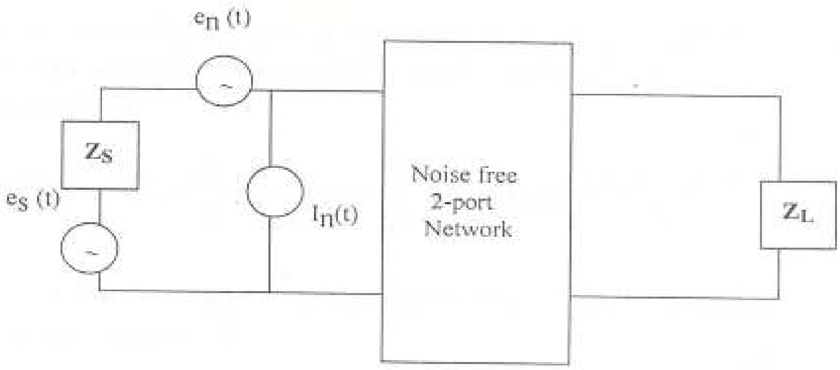
$$P_{n,out}(\tau) = \int_0^{\infty} S(W) \frac{dw}{2\pi} \qquad (2.40)$$

Noisy two-port networks

In analyzing the noise produced at the output of a linear 2-port network due to the internal noise sources, we can replace all of the internal noise sources by a series noise voltage generator e_n (t) and a hunt noise current generator i_n (t) at the input as shown in figure.



(a)



(b)

Fig.2.7 Equivalent input noise source for a noisy linear two port network

The total noise power at the output can be found by evaluating the noise output produced by $e_n(t)$, $i_n(t)$ and the thermal noise in the resistive component R_s of the source impedance. Two equivalent noise sources are needed at the input because if the input is short-circuited, that is, Zs = 0, the source $i_n(t)$ does not produce any output noise, yet the noisy 2 part does have a noisy output under short – circuit conditions at the input so a noise voltage source $e_n(t)$ is required. Similarly, under open-circuit conditions $e_n(t)$ does not produce any out noise, so a noise source $i_n(t)$ is needed to represent the equivalent input noise source under open-circuit conditions. The two noise source $e_n(t)$ & $i_n(t)$ are not completely independent since a part of $e_n(t)$ and $i_n(t)$ may arise from the same basic noise producing mechanism within the two-port network. Thus, in general, there is some cross-correlation between $e_n(t)$ and $i_n(t)$ with a resultant non-zero cross-power spectral density.

It is common practice to express the power spectral density associated with the two noise source $e_n(t)$ and $i_n(t)$ in from similar to that given by Equation (2.28) and Equation (230) for thermal noise. When this is done, flicker noise, which is low- frequency noise is not considered, since linear microwave amplifiers do not produce an output for low-frequency input signals, the neglect of flicker noise can thus be justified. Thus the spectral densities can be specified as follows:

For $e_n(t)$ $S_e(w)=4KTR_e$

For $i_n(t)$ $S_i(w)=4KTG_i$

And $2[S_{xr}(w)+jS_{xi}(w)]=4KT(Y_r+jY_i)$

Where R_e = Equivalent noise resistance.

G_i = equivalent noise conductance.

$Y_r + jY_i$ = complex equivalent noise impedance

.

In terms of the above spectral densities, the total noise input to the noise-free two-port network can be expressed, in a frequency band Δf as using equation (2.38)

$$P_{n,in} = KT\Delta f M + KT\Delta f \frac{R_e}{R_s} M + KT\Delta f \frac{G_i}{G_s} M + 2kT\Delta f \frac{(R_s Y_r + X_s Y_i)}{R_s} \qquad (2.41)$$

In this equation, the first term on the right, $KT\Delta f$, is the input thermal noise from the source resistance, R_s.

The output noise in Z_L in a frequency band Δf is obtained by multiplying the power gain g_p (w) of the network. The noise produced at the output termination by the equivalent source en (t) and in(t) placed at the input of the network I fully equivalent to that produced by the internal noise mechanisms in the real noisy two-port network.

2.6 LOW – NOISE AMPLIFIER DESIGN

In this section we will discuss noise figure and the design of an amplifier for minimum noise. It will be shown that there is an optimum noise. It will be shown that there is an optimum source impedance Zs (or source reflection co-efficient Γ_s) that will result in the lowest noise figure. It will be also introduced the constant noise-figure circles that can be plotted on the Γ_s plane and which will show in a pictorial way the increases in noise figure that occurs when the optimum source reflection coefficient cannot be used.

Noise figure

Referring figure (2a), the definition of noise figure (F) is [1, 2]:

$$F = \frac{signal - to - noise\ \ ratio\ \ at\ \ input}{signal - noise\ ration\ at\ output} \qquad (2.43)$$

Where the noise powers are those in a narrow frequency band (Δf). Based on this definition (later), the shot noise figure for the system shown in figure (2) my be obtained from equation (2.41) by dividing by KTΔfMR$_s$, as

$$F = 1 + \frac{R_e}{R_s} + \frac{G_{i_i}}{G_s} + 2\frac{R_s Y_r + X_s Y_i}{R_s} \qquad (2.44)$$

Where $\qquad G_s = \frac{R_s}{(R_s^2 + X_s^2)}$

This noise figure is seen to depend on the source impedance as well as on the noise parameters Re, G_i, Y_r and Y_i. The noise figure does not depend on the frequency bandwidth (Δf). However, the input signal- to- noise ratio will deteriorate if the amplifier bandwidth is greater than that required accommodate the signal.

The optimum source impedance that will minimize the noise figure is obtained by setting

$$\frac{\partial F}{\partial R_r} = 0, \frac{\partial F}{\partial X_s} = 0$$

And solving for R_s and X_s, we can easily find that

$$X_s = X_m = -\frac{Y_i}{G_i} \qquad (2.45a)$$

And $R_s^2 + X_s^2 = \frac{R_e}{G_i}$, which gives

$$R_s = R_m = \sqrt{\frac{R_e}{G_i} - \frac{Y_1^2}{G_i^2}} \tag{2.45b}$$

When these values of R_s and X_s are used in equation (2.44), we get the minimum noise figure, F_m.

Let us now replace Y_i by $(-X_mG_i)$ in Equation (2.44) and find out $(F-F_m)$.

$$F = 1 + \frac{R_e}{R_S} + \frac{G_i(R_S^2 + X_S^2)}{R_S} + 2\frac{R_S Y_r + X_S Y_i}{R_S}$$

$$F_m = 1 + \frac{R_e}{R_m} + \frac{G_i(R_m^2 + X_m^2)}{R_m} + 2\frac{R_m Y_r + X_m(-G_i X_m)}{R_S}$$

$$F - F_m = \frac{R_e}{R_S} + \frac{(R_S^2 + X_S^2)G_i}{R_S} + 2X_S(-X_m G_i) - \left[\frac{R_e}{R_m} + \frac{(R_m^2 + X_m^2)G_i}{R_m} + 2X_m(-X_m G_i)\right]$$

$$= \frac{1}{R_S}\left[R_e + (R_S^2 + X_S^2)G_i - 2X_m X_S G_i\right] - \frac{1}{R_m}[R_e + (R_M^2 + X_m^2)G_i - 2X_m^2 G_i]$$

$$= \frac{1}{R_S}\left[R_e + (R_S^2 + R_m^2 - R_m^2 - 2R_S R_m + 2R_S R_m)G_i + (X_s^2 + X_m^2 - X_m^2 - 2X_S X_m)G_i\right]$$
$$- \frac{1}{R_m}[R_e + (R_m^2 + X_m^2)G_i - 2X_m^2 G_i]$$

$$= \frac{1}{R_S}\left[R_e + (R_S - R_m)^2 G_i - (R_m^2 - 2R_S R^2{}_m - 2R_S R_m)G_i + (X_S - X_m)^2 G_i + (X_S - X_m)^2 G_i - X_m^2 G_i\right]$$
$$- \frac{1}{R_m}[R_e + R_m^2 G_i - X_m^2 G_i]$$

$$= \frac{1}{R_S}\left[R_e + (R_S - R_m)^2 G_i + (X_S - X_m)^2 G_i + 2R_S R_m G_i - R_m^2 G_i - X_m^2 G_i\right] - \frac{1}{R_m}[R_e + R_m^2 G_i - X_m^2 G_i]$$

$$= \frac{G_i}{R_S}\left[(R_S - R_m)^2 + (X_S - X_m)^2\right] + G_i + 2R_S R_m G_i - R_m^2 G_i - X_m^2 G_i\Big] - \frac{1}{R_m}[R_e + R_m^2 G_i - X_m^2 G_i]$$

Upon using $\frac{2R_S R_m G_i}{R_S} = \frac{2R_m^2 G_i}{R_m}$ and using equation. (2.45) we get

$$R_e - \left(R_m^2 + X_m^2\right)G_i$$

$$= R_e - \left(\frac{R_e}{G_i} - \frac{Y_i^2}{G_i^2}\right)G_i$$

$$Therefore, F = F_m + \frac{G_i}{R_S}\left[\left(R_S - R_m\right)^2 + \left(X_S - X_m\right)^2\right] \qquad (2.46)$$

Equation (2.46) is important because it determines the noise figure in terms of the minimum value F_m obtained when the optimum source impedance ($R_m + jX_m$) is used along with G_j. Transistor manufactures always specifies the minimum noise figure, Fm, the optimum source impedance ($R_m + jX_m$) or source reflection coefficient.

$$\Gamma_m = \frac{\left(R_m + jX_m - Z_C\right)}{\left(R_m + jX_m + Z_C\right)}$$

And the noise conductance G_i or noise resistance R_e.

If Re is given then by using $\left(R_m^2 + X_m^2\right) = \frac{R_e}{G_i}$, we can get

$$G_i = \frac{R_e}{R_S^2 + X_m^2} \qquad (2.47)$$

Then these data can be used to find noise figure F.

Constant noise figure circles

For the purpose of low- noise amplifier design, it is useful to plot constant noise figure circles on the source reflection-coefficient plane [2], since

$$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C}, \qquad \Gamma_m = \frac{Z_m - Z_C}{Z_m + Z_C}$$

The Equation (2.46) for noise figure can be written as follows:

$$F = F_m + \frac{G_i}{R_S}\left[(R_S - R_m)^2 + (X_S - X_m)^2\right]$$

$$or, F = F_m + \frac{G_i}{R_S}\left|Z_S - Z_m\right|^2$$

$$\Gamma_S = \frac{Z_S - Z_C}{Z_S + Z_C},$$

$$Or, \frac{1+\Gamma_S}{1-\Gamma_S} = \frac{Z_S - Z_C + Z_S + Z_C}{Z_S + Z_C - Z_S + Z_C} = \frac{Z_S}{Z_C}$$

When $Z_S = Z_m$, we can write

$$\frac{1+\Gamma_m}{1-\Gamma_m} = \frac{Z_m}{Z_C}$$

Therefore,

$$\left|Z_S - Z_m\right|^2 = \left|\frac{Z_S}{Z_C} - \frac{Z_m}{Z_C}\right|^2 \ X \ Z_C^2$$

$$= \left|\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right|^2 X \ \ Z_C^2$$

Therefore, $F - F_m = \frac{G_i}{R_S} Z_C^2 \left(\left|\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right|^2\right)$

Now, $$= \left|\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right|^2$$

$$= \left(\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right)\left(\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\mathrm{H}_m}{1-\Gamma_m}\right)^*$$

$$= \left(\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right)\left(\frac{1+\Gamma_S}{1-\Gamma_S}\right)^* - \left(\frac{1+\Gamma_m}{1-\Gamma_m}\right)^*$$

$$= \left(\frac{1+\Gamma_S}{1-\Gamma_S} - \frac{1+\Gamma_m}{1-\Gamma_m}\right)\left(\frac{1+\Gamma_S{}^*}{1-\Gamma_S{}^*} - \frac{1+\mathrm{H}^*{}_m}{1-\Gamma^*{}_m}\right)$$

$$= \frac{2(\Gamma_s - \Gamma_m)2(\Gamma_s^* - \Gamma_m^*)}{(1-\Gamma_s)(1-\Gamma_m)(1-\Gamma_s^*)(1-\Gamma_m^*)}$$

$$= \frac{4\left|\Gamma_s - \Gamma_m\right|^2}{\left|1 - \Gamma_m\right|^2 \left|1 - \Gamma_m\right|^2}$$

Therefore $F - F_m = \frac{G_i}{R_s} Z_c^2 \frac{4\left|\Gamma_s - \Gamma_m\right|^2}{\left|1-\Gamma_m\right|^2\left|1-\Gamma_m\right|^2}$

Since, $\frac{Z_s}{Z_c} = \frac{1+\Gamma_s}{1-\Gamma_s}$

Therefore, $\frac{R_s}{Z_c} = R_e\left(\frac{1+\Gamma_s}{1-\Gamma_s}\right)$

Or, $\frac{2R_s}{Z_c} = 2R_e\left(\frac{1+\Gamma_s}{1-\Gamma_s}\right)$

So, we can write

$$\frac{2R_s}{Z_c} == \frac{Z_S + Z_S^*}{Z_C} = 2R_e\left(\frac{1+\Gamma_s}{1-\Gamma_s}\right)$$

$$= 2R_C \frac{(1+\Gamma_S)(1-\Gamma_S)^*}{\left|1-\Gamma_S\right|^2} = \frac{2\left(1-\left|\Gamma_S\right|^2\right)}{\left|1-\Gamma_S\right|^2}$$

Therefore, $F - F_m = \frac{G_i}{R_s} Z_c^2 \frac{4\left|\Gamma_s - \Gamma_m\right|^2}{\left|1-\Gamma_m\right|^2\left|1-\Gamma_m\right|^2}$

$$= G_i \frac{\left|1-\Gamma_S\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)} Z_C \frac{4\left|\Gamma_S - \Gamma_M\right|^2}{\left|1-\Gamma_S\right|^2\left|1-\Gamma m\right|^2}$$

$$= 4G_i Z_C \frac{\left|\Gamma_S - \Gamma_M\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)\left|1-\Gamma m\right|^2}$$

Let $G_i Z_c = G_I$

Therefore, $F - F_m = 4G_I \frac{\left|\Gamma_s - \Gamma_m\right|^2}{\left|1-\Gamma_m\right|^2\left|1-\Gamma_m\right|^2}$

A similar expression using $R_N = R_e / Z_c$ is,

$$F - F_m = 4R_N \frac{\left|\Gamma_s - \Gamma_m\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right)\left|1 - \Gamma_m\right|^2}$$

We now introduce the parameter N_i defined as,

$$N_i = \frac{\left(F_i - F_M\right)\left|1 - \Gamma_M\right|^2}{4G_i} = \frac{\left|\Gamma_s - \Gamma_m\right|}{\left(1 - \left|\Gamma_S\right|^2\right)}$$

Where N_i, corresponding to a chosen value F_i for F.

Therefore, $N_i = \dfrac{\left|\Gamma_S - \Gamma_m\right|^2}{1 - \left|\Gamma_S\right|^2} = \dfrac{\left(\Gamma_S - \Gamma_m\right)\left(\Gamma_S^* - \Gamma_m^*\right)}{1 - \Gamma_S \Gamma_S^*}$

$$= \frac{\Gamma_S \Gamma_S^* - \Gamma_S \Gamma_m^* - \Gamma_S{}^* \Gamma_m - \Gamma_m \Gamma_m^*}{1 - \Gamma_S \Gamma_s^*}$$

By comparing it with Equation (1.1), it may be seen that it is the equation of a circle described in the Γs plane. The center of the circle is located at

$$\Gamma_{sf} = \frac{\Gamma_m}{1 + N_i}$$

And the radius of the circle is given by,

$$R_f = \frac{\sqrt{N_i^2 + N_i\left(1 - \left|\Gamma_m\right|^2\right)}}{1 + N_i}$$

We can plot the circles for various values of Ni (determined by Equation (2.49) for chosen values of Fi),. Each circle shows the values Γs that can be used in order to get a noise figure equal to Fi, If Γs = Γm is a stable point, then the amplifier can be designed to give a minimum noise figure Fm.

Usually, we ca choose the load ZL so that Γs=Γm gives a stable design. However sometimes the input VSWR is too high if Γs=Γm. In such a case a choice for Γs on a constant noise figure circle with Fi > Fm would be used in order to obtain a better input VSWR.

Chapter 3

RESULT AND DISCUSSION

3.1 INTRODUCTION

In chapter 2, design methodology of the low-noise microwave amplifier is presented. Thereon the calculations are shown on the basic of a given set of value for noise figure and power gain. In this chapter, the elements of s- parameters are determined on the basics of required power gain and noise figure. Further, the performance of the amplifier is obtained on the variation of practical range of S-parameters.

3.2 RESULTS AND DISCUSSIONS

3.2.1 Calculation of scattering matrix from given amplifier specifications

This section details the attempt of low noise amplifier design using BJT, which is also absolutely stable based on the specifications given below:

Noise figure = 4.5 dB, that is, Minimum noise figure, fm = 2.818

Operating frequency = 1 GHz

Power gain = 15 dB, that is G_p= 31.62

Source impedance = 100 ohms, that is $Z_s = Z_m$ (optimum source impedance)

To obtain the scattering parameters for the amplifier and the given performance parameters, the initial value of the scattering matrix parameters are chosen as:

$S_{11} = 0.36\angle 148^\circ$, $S_{12} = 0.11\angle 42^\circ$, $S_{21} = 1.57\angle 27^\circ$, $S_{22} = 0.67\angle -64^\circ$

Calculation of Stability Parameter (K)

The stability parameters is calculated using Equation (2.21 a) as

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$= 1.24 > 1$

Where the determinant of the S – Matrix, $\Delta = S_{11}S_{22} - S_{12}S_{21}$, is calculated as

$=0.2412\angle 84^\circ - 0.1727 \angle 69^\circ$

$=0.25+j0.239 - 0.061-j0.161$

$= 0.086\angle 114.31^\circ$

Since, the value of K is obtained as more than 1, the device is absolutely stable.

Therefore, the assumed value of S- parameters is the justified values.

Calculation of Normalization power gain (g_p)

The maximum power gain is obtained using equation (2.15) as

$$G_{P\max} = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2 - 1}\right)$$

= 14.27 X 0.506

=7.23

But the desired G_p is 31.62. So to increase the G_p from 7.23 to 31.62, the value of $\left|\frac{S_{21}}{S_{12}}\right|$ is increased to a greater value such that $|S_{12}S_{21}|$ remains same. So, Δ will be same and thereby maintains a constant value of K. In this design, the value of $\left|\frac{S_{21}}{S_{12}}\right|$ is increased by a factor of 4.37 (that is, 31.62/ 7.23), which however, maintains the same value of $|S_{12}S_{21}|$ at 0.1727 as earlier.

Therefore, the new values of $[S_{12new}]$ and $[S_{21new}]$ is found as

$[S_{21new}] = 3.28$

$[S_{12new}] = 0.052$

So, the scattering matrix parameters now has been modified to,

$S_{11} = 0.36\angle 148^\circ$, $S_{12} = 0.052\angle 42^\circ$, $S_{21} = 3.28\angle 27^\circ$, $S_{22} = 0.67\angle -64^\circ$ Using equation (2.22), the normalized gain is calculated as

$$g_p = \frac{31.62}{(3.28)^2} = 2.939$$

Calculation of Reflection Coefficients

Since, the device is absolutely stable, so conjugate impedance matching is considered. This leads to $Z_s = Z_{in}^*$, $Z_L = Z_{out}^*$, and hence, $\Gamma_s = \Gamma^*_{in}$ and $\Gamma_L = \Gamma^*_{out}$, and $M_s = M_L = 1$.

Using equation (2.13), the value of

$$\Gamma_S = \Gamma_{SM} = \frac{1}{2B_1}\left[A_1 \pm \sqrt{A_1^2 - 4|B_1|^2}\right] \tag{2.13a}$$

$$\Gamma_L = \Gamma_{LM} = \frac{1}{2B_1}\left[A_1 \pm \sqrt{A_1^2 - 4|B_1|^2}\right] \qquad (2.13\text{ b})$$

Where,

$$A_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 = 1 + (0.36)^2 - (0.67)^2 - (0.086)^2 \quad = 0.6733$$

$$A_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 = 1 + (0.67)^2 - (0.36)^2 - (0.086)^2 \quad = 1.312$$

$$B_1 = S_{11} - \Delta S_{22}^* = 0.36\angle 148^\circ - 0.086\angle 114.31^\circ 1067\angle 64^\circ = 0311\angle 142.83^\circ$$

$$B_2 = S_{22} - \Delta S_{11}^* = 0.67\angle -64^\circ - 0.086\angle 114.31^\circ \times 0.36\angle -148^\circ = 0.64\angle -65.3^\circ$$

Therefore,

$$\Gamma_S = \Gamma_{SM} = \frac{1}{2X0.311\angle 142.83^\circ}\left(0.6733 - \sqrt{0.4533 - 4(0.311)^2}\right)$$

$$\Gamma_L = \Gamma_{LM} = \frac{1}{2X0.64\angle -65.3^\circ}\left(1.312 - \sqrt{1.721 - 4X0.4}\right)$$

As conjugate impedance matching is used, therefore, the input and output VSWR = 1

Further, the value of minimum reflection coefficient from the given value of $Z_s = 100\ \Omega = Z_m$, is found as

$$\Gamma_m = \frac{Z_m - Z_C}{Z_m + Z_C}$$

$$= \frac{100 - 50}{100 + 50}$$

$$= 0.33$$

Determination of Noise Figure (F)

Assuming the noise resistance to be 20 Ω, and characteristics impedance of the input and output of the transmission line is 50 Ω, the noise figure is obtained by equation (2.48b) as

$$F = F_m + 4R_N \frac{|\Gamma_s - \Gamma_m|^2}{|1 + \Gamma_m|^2\left(1 - |\Gamma_s|^2\right)}$$

$$= 2.818 + \frac{4x0.4|-05.32 - j0.4 - 0.33|^2}{|1 + 0.33|^2\left(1 - 0.668^2\right)}$$

= 4.30 = 6.335 dB.

This is 1.835 dB greater than the given minimum value.

It is desirable to have a lower noise figure and yet without sacrificing any gain. But increase in VSWR at both input and the output of the amplifier may be accepted.

3.3 Modified Design

Therefore, a design using g_p = 2.9 (slightly less than the desired value) is attempted which is also very close to its maximum value, g_p (max).

For the constant normalized power gain circle for g_p = 2.9, the center and radius are calculated by the equations (2.24a) and (2.24b) respectively as,

$$\Gamma_{Lg} = \frac{(S_{22}^* - \Delta^* S_{11}) g_p}{\left(|S_{22}|^2 - |\Delta|^2\right) g_p + 1}$$

Therefore,

$$\Gamma_{Lg} = \frac{(0.67\angle 64^\circ - 0.086\angle -114.31^\circ \times 0.36\angle 148^\circ) \times 2.9}{[(0.67)^2 (0.086)^2 \times 2.9 + 1}$$

$$=0.811 \angle 65.38^\circ$$

$$= 0.337 + j0.737$$

and $$R_{Lg} = \frac{\sqrt{1 - 2Kg_p|S_{12}S_{21}| + g_p^2|S_{12}S_{21}|^2}}{\left|\left(|S_{22}|^2 - |\Delta|^2\right) g_p + 1\right|}$$

$$= \frac{\sqrt{1 - 2x1.24x2.9x0.1727 + 2.9^2 x0.1727^2}}{(0.67^2 - 0.086^2)2.9 + 1}$$

$$= \frac{0.093}{2.23}$$

$$= 0.04$$

The values of Γ_L = constant circle generate a set of values for Γ_{in} by the formula given as,

$$\Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{\Delta\Gamma_L - S_{11}}{S_{22}\Gamma_L - 1}$$

Which provides a circle with Γ_{in}* values. The center and radius of the circle are calculated as follows:

$$\Gamma_{in,c}^* = \frac{R_{Lg}^2 S_{22}\Delta^* + (S_{22}\Gamma_{Lg} - 1)(S_{11}^* - \Delta^*\Gamma_{Lg}^*)}{|R_{Lg}S_{22}|^2 - |\ S_{22}\Gamma_{Lg} - 1\ |^2}$$

Or,

$$0.04^2 \text{ x}0.67\angle\ 64^\circ \text{ x } 0.086\angle\text{-}114.31^\circ$$

$$\Gamma^*_{in,C} \quad = \frac{+(0.67\angle-64^\circ \times 0.811\angle 65.38^\circ -1)(0.36\angle-148^\circ - 0.086,-114.31^\circ \times 0.811\angle-65.38^\circ)}{(0.04\times 0.67)^2 - \left|0.67\angle-64^\circ \times 0.811\angle 65.38^\circ -1\right|^2}$$

$$= \text{-}0.513 \text{ - j}0.398$$

$$R_{in} = \frac{|S_{12}S_{21}|R_{Lg}}{|\ |R_{Lg}S_{22}|^2 - |\ S_{22}\Gamma_{Lg} - 1\ |^2\ |}$$

$$R_{in} = \frac{0.1727x0.04}{\left|(0.04x0.67)^2 - \left|(0.67\angle-64^\circ 0.811\angle 65.38^\circ -1\right)^2}$$

$$= 0.033$$

The content noise figure circle for F = 3.162, that is, 5 dB, (which is generally assumed greater than desired value) may be drawn having the following centers and radius given by the equations (2.50) as

$$\Gamma_{sf} = \frac{\Gamma_m}{1+N_i} \tag{2.50a}$$

$$Rf = \frac{\sqrt{N_i^2 + N_i\left(1-|\Gamma_m|^2\right)}}{1+N_i} \tag{2.50b}$$

$$\therefore \Gamma_{sf} = \frac{0.33}{1+N_i}$$

Where $N_i = \dfrac{(F_i - F_m)|1+\Gamma m|^2}{1+N_i}$ (2.49)

$$= \frac{(3.162-2.818)|1+0.33|^2}{4x0.4}$$

$$= 0.380$$

Therefore,

$$\Gamma_{sf} = \frac{0.33}{1+0.380} = 0.239$$

$$R_f = \frac{\sqrt{0.38^2 + 0.38(1-0.33^2)}}{1+0.38}$$

$$= 0.53$$

Here it is insisted on having a g_p of 2.9 and if a good input VSWR is desired, then on needs to accept some increase in noise figure [2]. If one accepts a noise figure 0.5 dB greater than F_m then for the best input VSWR, Γ_s is chosen on lie on the F = Fm + 0.5 dB constant noise figure circle and as close as possible to the Γ_{in}^* circle for gp = 2.9. These points (which are closest to each-other on the constant noise figure circle and Γ_{in}^* circle) are obtained from the smith chart as shown in the figure (3.1) as

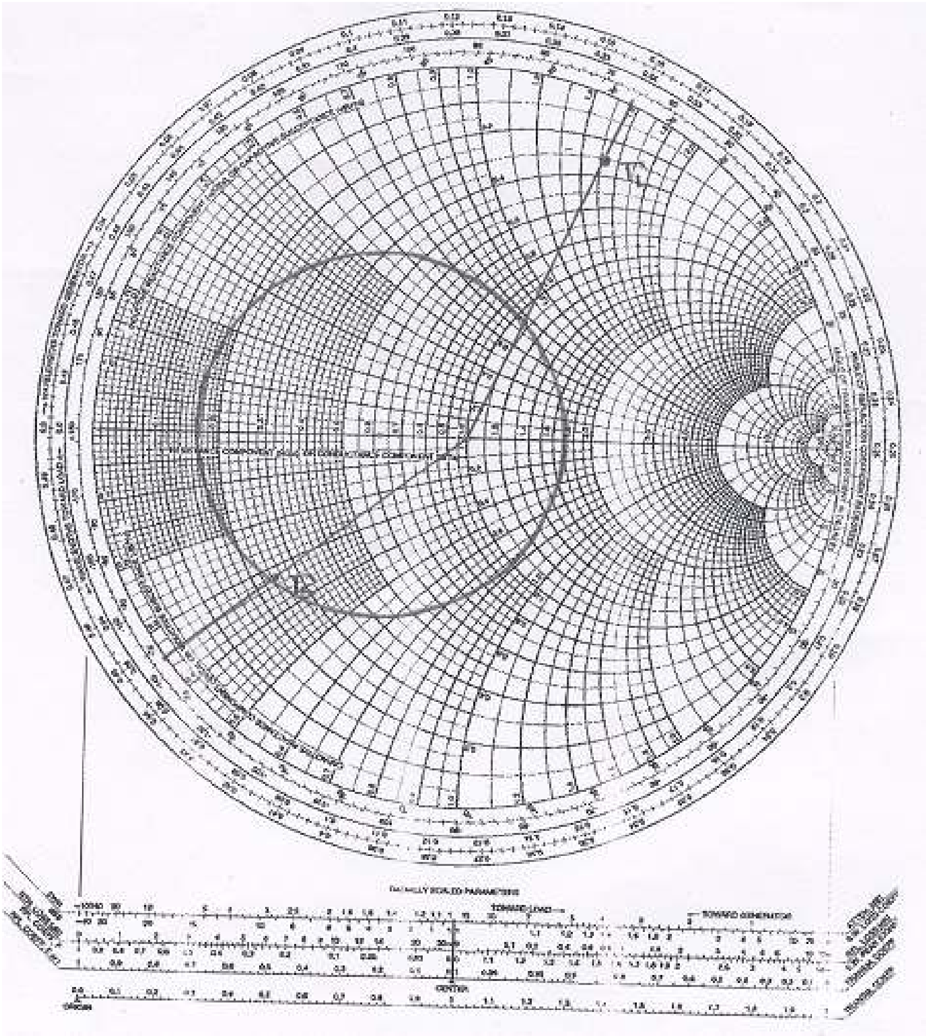


Figure 3.1: Smith chart for Γ_s

$$\Gamma_s = 0.649\angle -138^\circ = -0.4823 - j0.4343$$

Then the value of Γ_{in}^* circle which is close to the point Γ_s is

$$\Gamma_{in}^* = 0.649\angle -138^\circ = -0.4823 - j0.4343$$

But for conjugate impedance matching,

$$\Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\text{Or, } 0.649\angle -138^\circ = \frac{0.36\angle 148^\circ - 0.086\angle 114.31^\circ \Gamma_L}{1 - 0.67\angle -64^\circ \Gamma_L}$$

$$\text{Or, } \Gamma_L = 0.8108\angle 60.8248^\circ = 0.3953 + j0.708$$

Determination of Modified VSWR

The above choices result in an input VSWR equal to,

$$\text{VSWR} = \frac{1 + \sqrt{1 - M}}{1 - \sqrt{1 - M}}$$

Where mismatch at input, M

$$= \frac{\left(1 - |\Gamma_{in}|^2\right)\left(1 - |\Gamma_s|^2\right)}{|1 - \Gamma_s \Gamma_{in}|^2}$$

$$= \frac{(1 - 0.649^2)(1 - 0.649^2)}{\left|1 - 0.649\angle -138^\circ 0.649\angle 138^\circ\right|^2}$$

Therefore, input VSWR = 1: **same as before.**

Similarly, output VSWR may be calculated as:

$$\Gamma_s = 0.4823 - j0.4343$$

$$\Gamma_{out} = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$= \frac{0.67\angle 64^\circ - 0.0886\angle 113.9067^\circ 0.649\angle -138^\circ}{1 - 0.36\angle 148^\circ 0.649\angle -138^\circ}$$

$$= 0.8170 \angle 65.4195^0$$

Therefore, output VSWR,

$$= \frac{1 + |\Gamma_{out}|}{1 - |\Gamma_{out}|}$$

$$= \frac{1 + 0.8170}{1 - 0.8170}$$

$$= 9.9284$$

Therefore, it is seen that although there is no change is occurred in $VSWR_{in}$, large change is observed in $VSWR_{out,}$ to maintain a constant gain and noise figure.

3.4 DESIGN PARAMETERS AS A FUNCTION OF SCATTERING MATRIX COMPONENTS

In this section, the components of the scattering matrix are varied to see the variation of performance parameters, such as, $VSWR_{out,}$ $VSWR_{in}$, noise figure (F) and normalized gain (g_p), of the microwave amplifier. To observe the variation, only a single component of the scattering matrix is varied, while others remain constant. The following discussion pertains to above.

Case1: When $|S_{11}|$ is varied

Figure 3.2 shows the variation of amplifier performance parameters, such as, gain, noise figure, $VSWR_{in}$, $VSWR_{out}$,as a function of $|S_{11}|$. As may be seen from the figure, $VSWR_{out}$ is increasing rapidly among the curves with the increase in $|S_{11}|$. For example, the value of $VSWR_{out}$, rises from a value of 9.0827 to 11.6748 hen $|S_{11}|$ is varied from 0.28 to 0.4, and further increases to 15.07 when $|S_{11}|$ = 0.44.

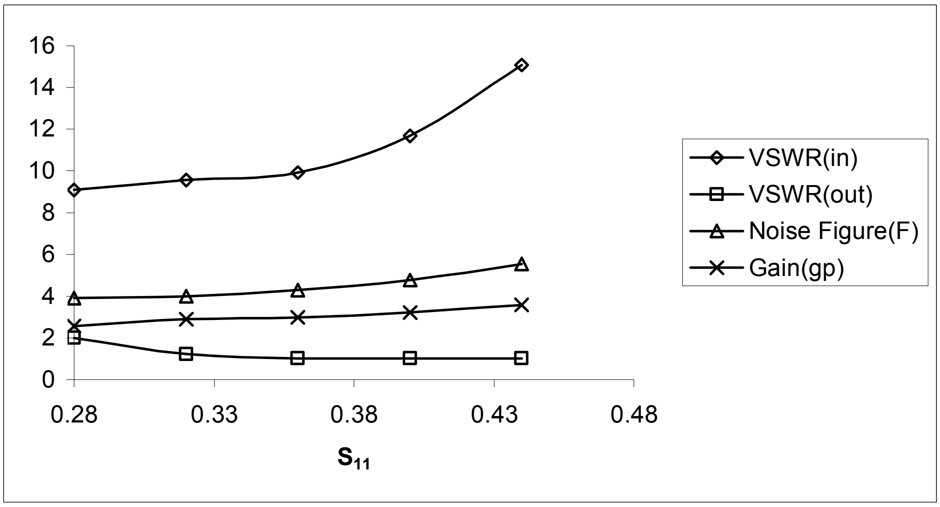


Figure 3.2: Variation of Amplifier Performance Parameters with $|S_{11}|$ (for $|S_{12}|$ = 0.052, $|S_{21}|$=3.28 and $|S_{22}|$=0.67)

As may be seen $VSWR_{in}$, decreases very slowly from a value of 1.9950 to 1, when $|S_{11}|$ is varied from 0.34 to 0.4 and remains same throughout when $|S_{11}|$ takes values greater than 0.4 onwards.

It may also be seen from the figure that the values of gain (g_p) and that of noise figure (F) increases rather slowly as compared with $VSWR_{out}$ from values of 2.5768 and 3.8992 respectively when $|S_{11}|$=0.28 to an intermediate values of 3.5700, 5.5408, when $|S_{11}|$=0.4 and further increases to values 3.9028 and 6.1864 respectively when $|S_{11}|$ = 0.44.

The detailed results of above changes are presented in table 3.1

Table 3.1: Calculated design parameters as a function of $|S_{11}|$

$\|S_{11}\|$	**$VSWR_{in}$**	**$VSWR_{out}$**	**Noise figure, F**	**Normalized gain, g_p**
0.28	1.9950	9.0827	3.8992	2.5768
0.32	1.2200	9.5585	4.0012	2.9028
0.36	1.0	9.9284	4.30	2.989
0.40	1.0	11.6748	4.7590	3.2216
0.44	1.0	15.0700	5.5408	3.5700

Case 2: when $|S_{12}|$ is varied

The variation of $VSWR_{out}$ is more when $|S_{12}|$ is varied. However, it increase rather slowly from 5.8806 to an intermediate value of 7.4112 when $|S_{12}|$ is varied from 0.028to 0.040, but this it increase rapidly to a value 12.8480when $|S_{12}|$ is varied to 0.058.

The variation of $VSWR_{in}$ is not monotonic .It increases first from a value of 1.0997 to 1.3228 when $|S_{12}|$ is increased from 0.028 to 0.040, and then decreases to a value of 1 when $|S_{12}|$ is further increased to 0.058.

The noise figure (F) increases from a value of 3.0128 to a value 3.6829 when $|S_{12}|$ is varied from 0.028 to 0.040 but after this value if $|S_{12}|$ is increased then variation of F is more as can be seen from the Figure 3.3.The value of F increases from 3.6829 to 4.7850 when $|S_{12}|$ is increased from 0.040 to 0.058. The increase of gain (g_p) is rather slow but monotonically as can be seen from the figure. It values changes from 2.1248 to 3.5128 as $|S_{12}|$ takes value in the range of 0.028 to 0.058.

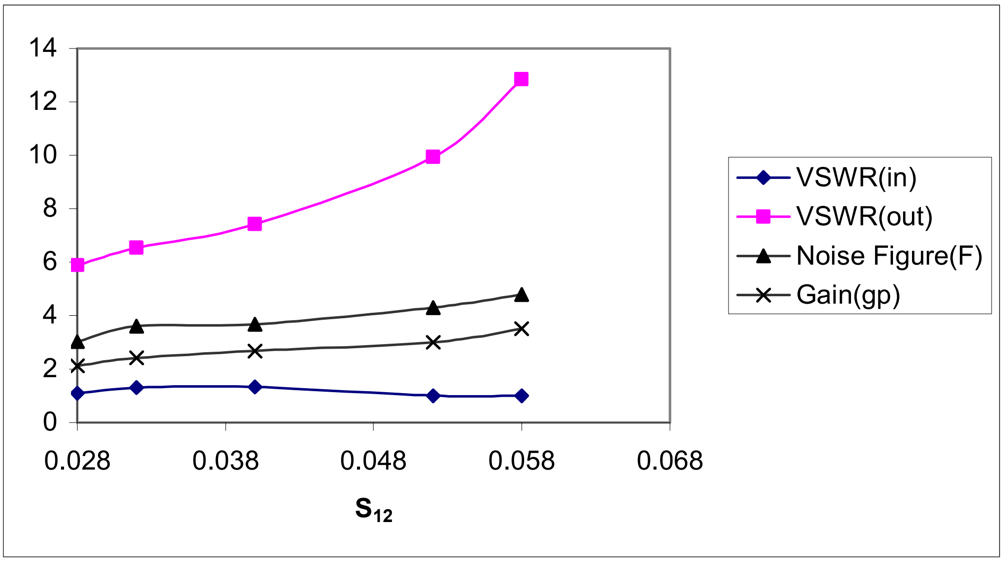


Figure 3.3: Variation of Amplifier Performance Parameters with $|S_{12}|$ **(for** $|S_{11}|$=0.36, $|S_{21}|$=3.28 and $|S_{22}|$=0.67)

The detailed variation of amplifier performance partakers with $|S_{12}|$ is presented in table3.2

Table 3.2: Calculated design parameters as a function of $|S_{12}|$

$\|S_{12}\|$	**$VSWR_{in}$**	**$VSWR_{out}$**	**Noise figure, F**	**Normalized gain, g_p**
0.028	1.0997	5.8806	3.0128	2.1248
0.032	1.2947	6.5300	3.6016	2.4083
0.040	1.3228	7.4112	3.6829	2.6822
0.052	1.0	9.9248	4.30	2.9890
0.058	1.0	12.8480	4.7850	3.5128

Case 3: when $|S_{21}|$ is varied

In this case, it may be seen from figure 3.4 that $VSWR_{out}$ increases when $|S_{21}|$ is increased but in a lesser rate compared to that in case l and case 2. It increases from 9.4050 to 10.9867 when $|S_{21}|$ is varied from 3.20 to 3.45, monotonically**.**

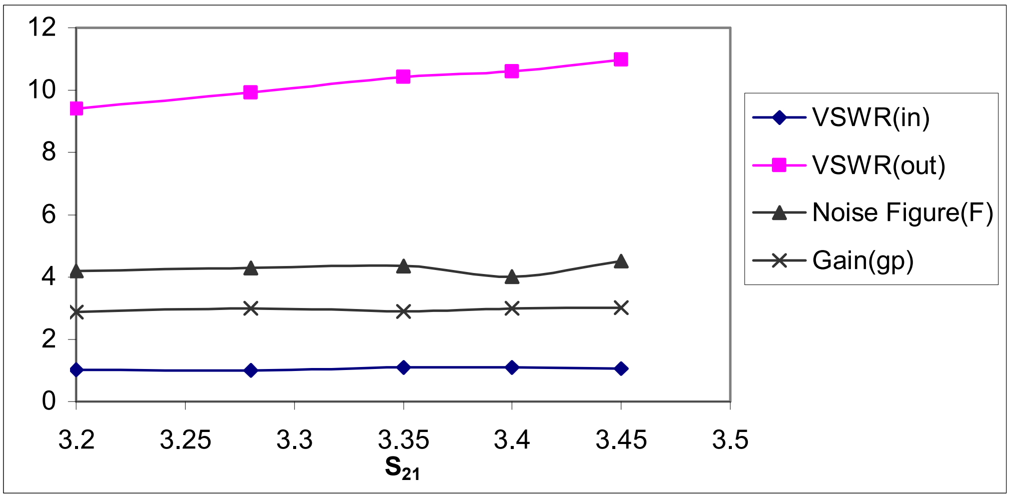


Figure 3.4: Variation of Amplifier Performance Parameters with $|S_{21}|$

(for $|S_{11}|$=0.36$|S_{12}|$=0.052 and $|S_{22}|$=0.67**)**

However, the values of $VSWR_{in}$, noise figure (F) and gain (g_p) remains almost constant with variation of $|S_{21}|$. As may be seen that is decreasing from a value of 1.099 to .1.0508, noise figure (F) is increasing from 4.1729 to 4.5126 and g_p is increasing from 2.8826 to 3.0056 when $|S_{21}|$ is varied from 3.20 to 3.45. The detailed variation of amplifier performance parameter with $|S_{21}|$ is presented in table 3.3.

Table 3.3: Calculated design parameters as a function of $|S_{21}|$

$\|S_{21}\|$	**$VSWR_{in}$**	**$VSWR_{out}$**	**Noise figure, F**	**Normalized gain, g_p**
3.20	1.0990	9.4050	4.1927	2.8826
3.28	1.0	9.9248	4.30	2.989
3.35	1.0952	10.4128	4.3570	2.90
3.40	1.0911	10.5988	4.0211	3.0
3.45	1.0508	10.9867	4.5126	3.0056

Case 4: when $|S_{22}|$ is varied

Figure 3.5 shows the variation of amplifier performance parameter, such as, gain, noise figure, $VSWR_{in}$, $VSWR_{out}$ as a function of $|S_{22}|$. As may be seen from the figure, is increasing exponentially with the increase in $|S_{22}|$. For example, the value of $VSWR_{out}$ rises from a value of 4.3515 when $|S_{22}|$ = 0.52 to 9.9284 when $|S_{22}|$ = 0.67, which further rises to 23.8249 when $|S_{22}|$=0.75.

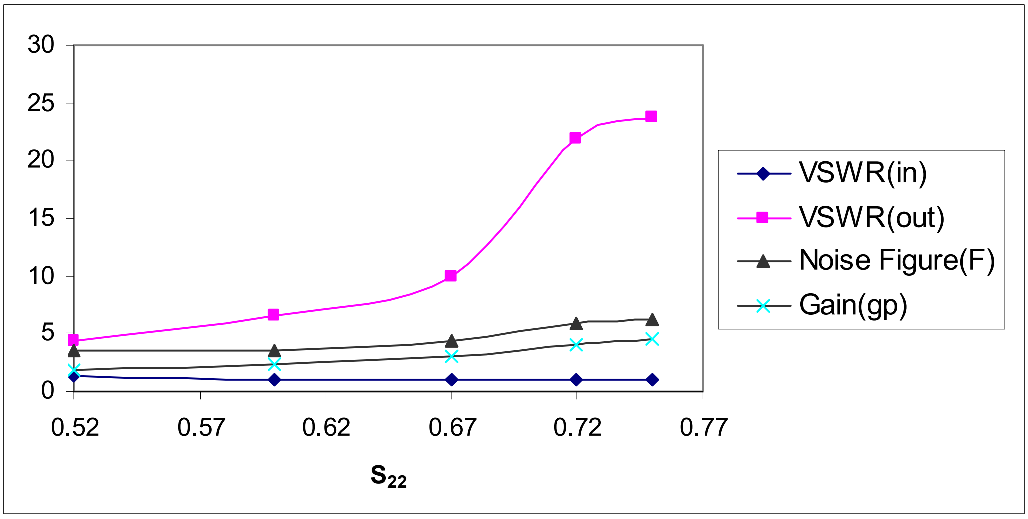


Figure 3.5: Variation of Amplifier performance Parameters with $|S_{22}|$ (for $|S_{11}|$=0.36 $|S_{12}|$=0.052 and $|S_{21}|$= 3.28)

Further, it may be seen from the figure that $VSWR_{in}$ decreases very slowly from a value of 1.2822, when $|S_{22}|$ = 0.52 to an intermediate value of 1.0126, when $|S_{22}|$=0.60, and then to a value 1 when $|S_{22}|$= 0.75.

It may also be seen from the figure that the values of gain (g_p) and the that of noise figure (F) increase slowly from values 3.6221 and 1.8192 when $|S_{22}|$ = 0.52 to an intermediate value of 4.30 and 2.989 when $|S_{22}|$ = 0.67 and further increases to values 6.2811 and 4.5238 respectively when $|S_{22}|$ = 0.75

Table 3.4: Calculated design parameters as a function of $|S_{22}|$

$\lvert S_{22}\rvert$	$VSWR_{in}$	$VSWR_{out}$	Noise figure, F	Normalized gain, g_p
0.52	1.2822	4.3515	3.6221	1.8192
0.60	1.0126	6.5316	3.5027	2.3222
0.67	1.0	9.9248	4.30	2.989
0.72	1.0	21.8595	5.9043	4.0918
0.75	1.0	23.8248	6.2811	4.5238

3.5 ADVANCE DESIGN SYSTEM

Advance Design system is used as the main tool for simulation and optimization of amplifier circuits. Hence a single stage microwave amplifier has been simulated by the Advance Design System. The amplifier has an n-p-n Bipolar Junction Transistor, which has following specifications, the base current is 60 μA and the collector-emitter voltage is 2.7V.

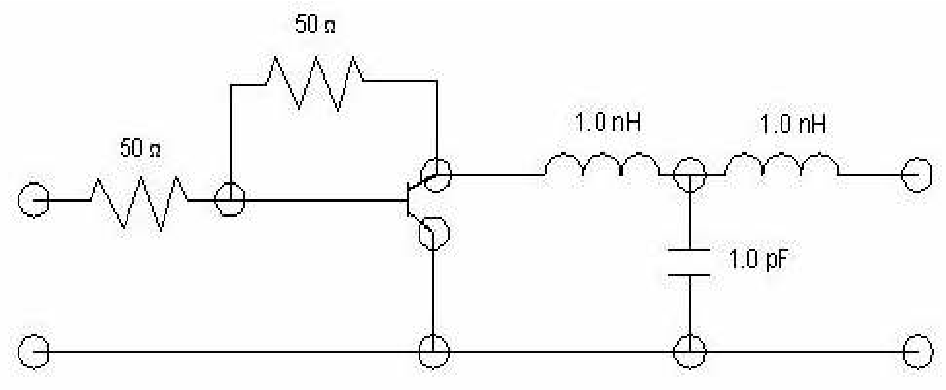


Figure 3.6: Circuit diagram of a single stage microwave amplifier

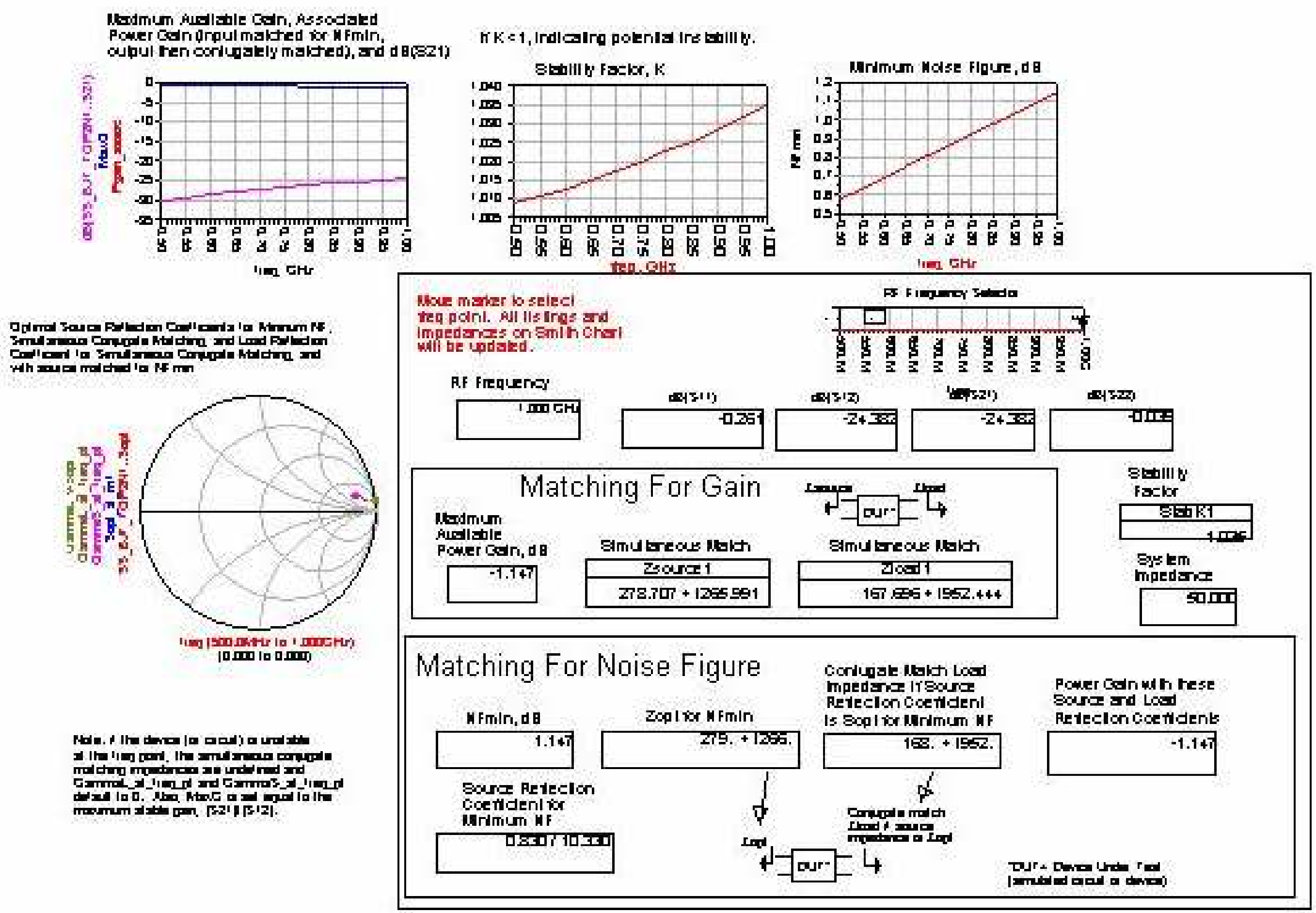


Figure 3.7: Simulation result for noise figure and stability of the single stage microwave amplifier

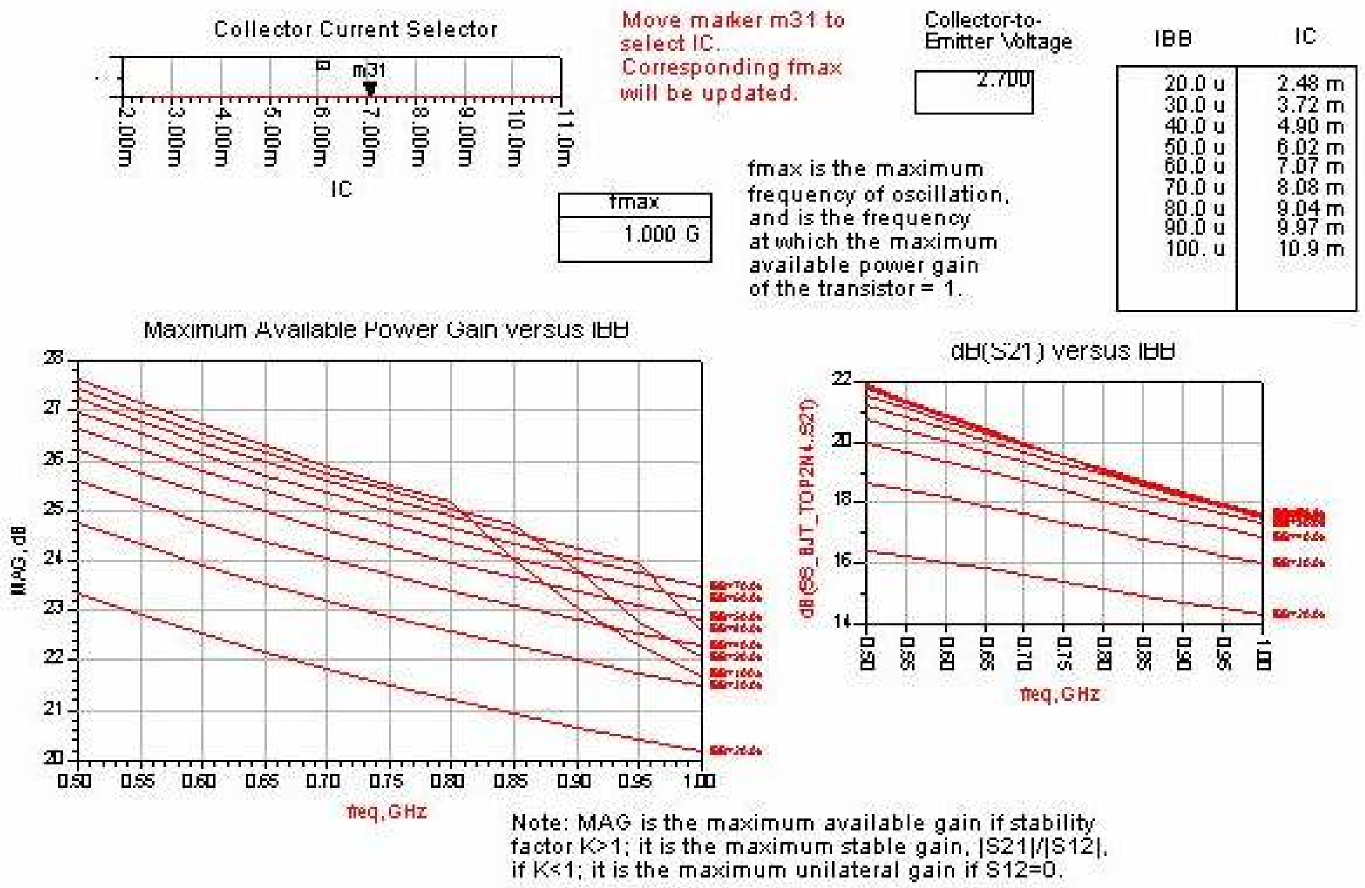


Figure 3.8: Simulation result for maximum available power gain Vs bias

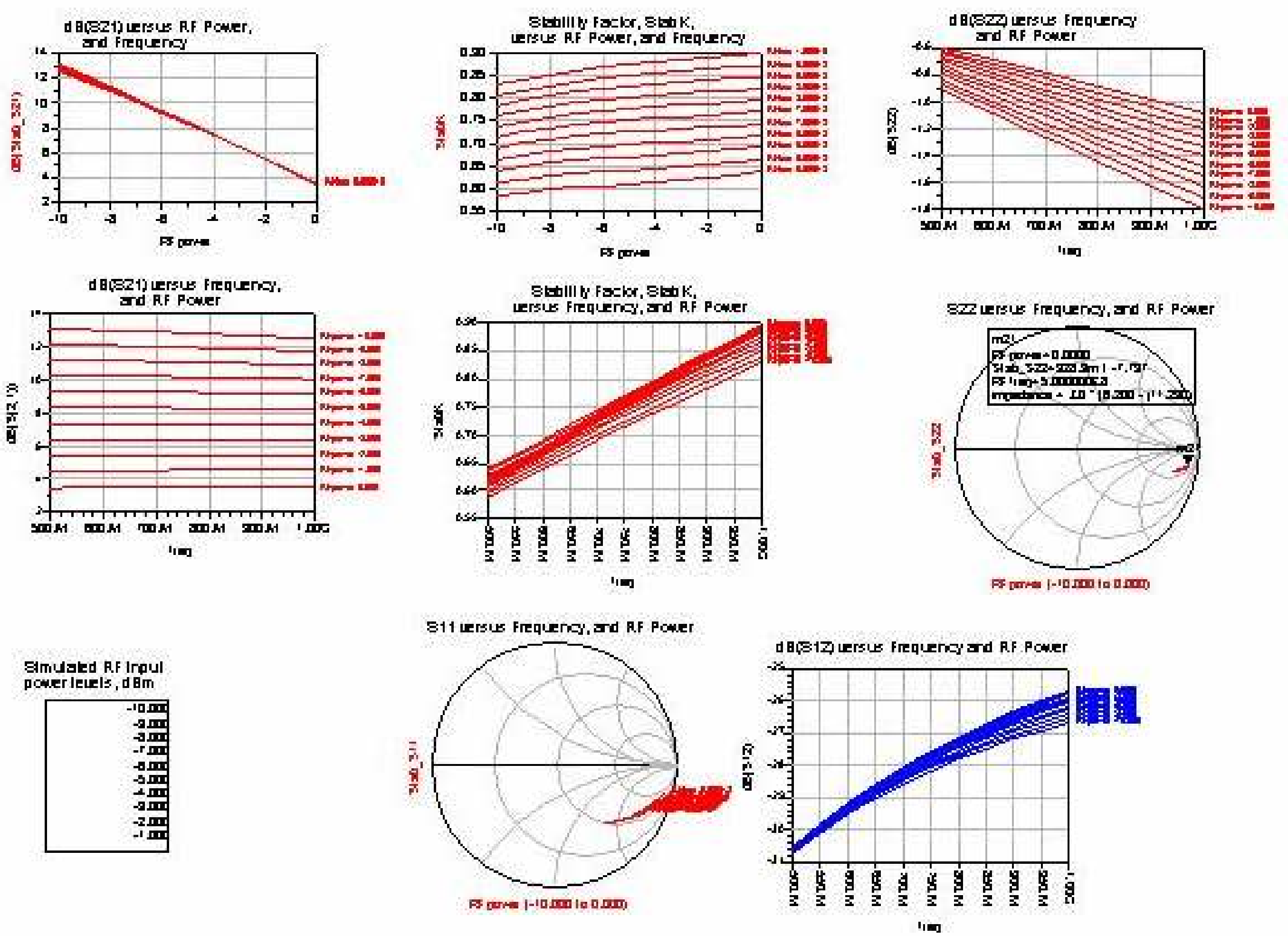


Figure 3.9: Simulation result for S parameters, stability Vs frequency and RF power

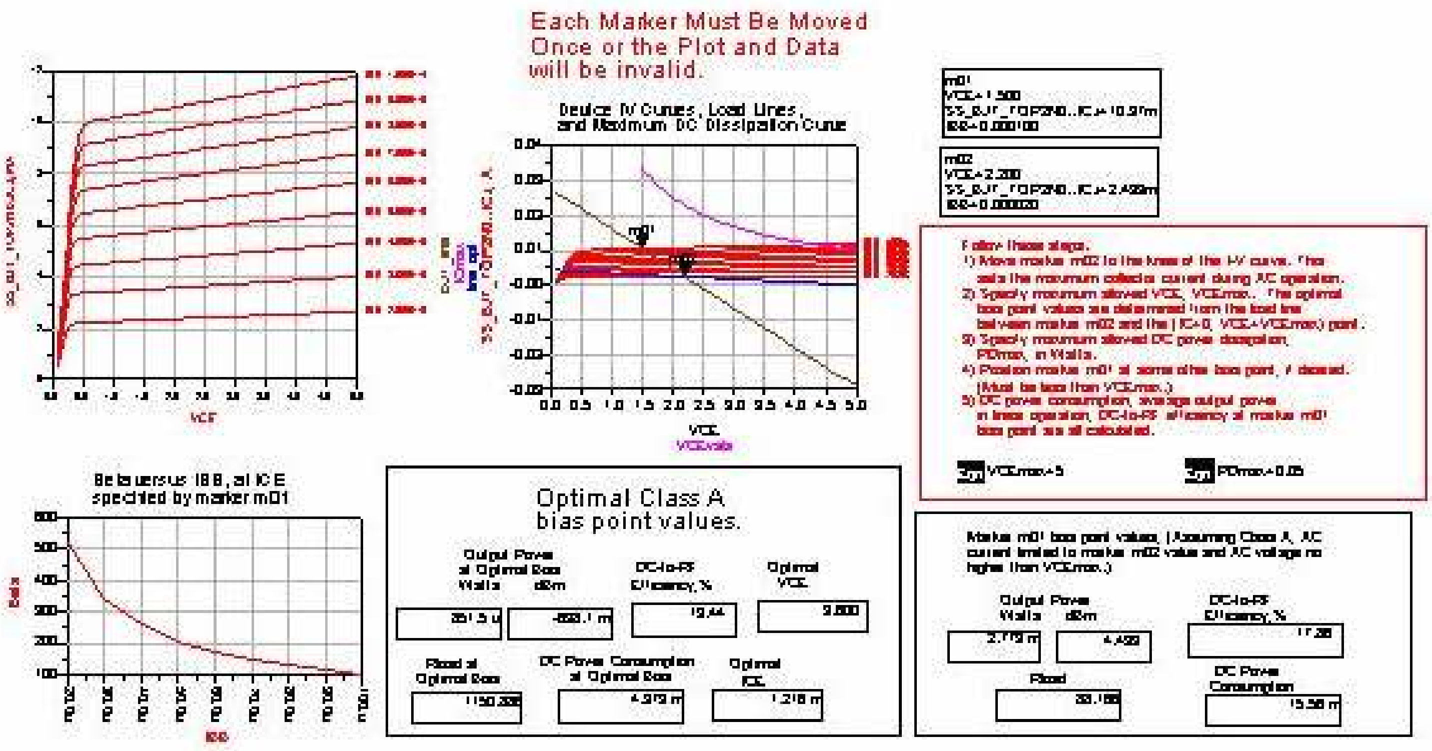
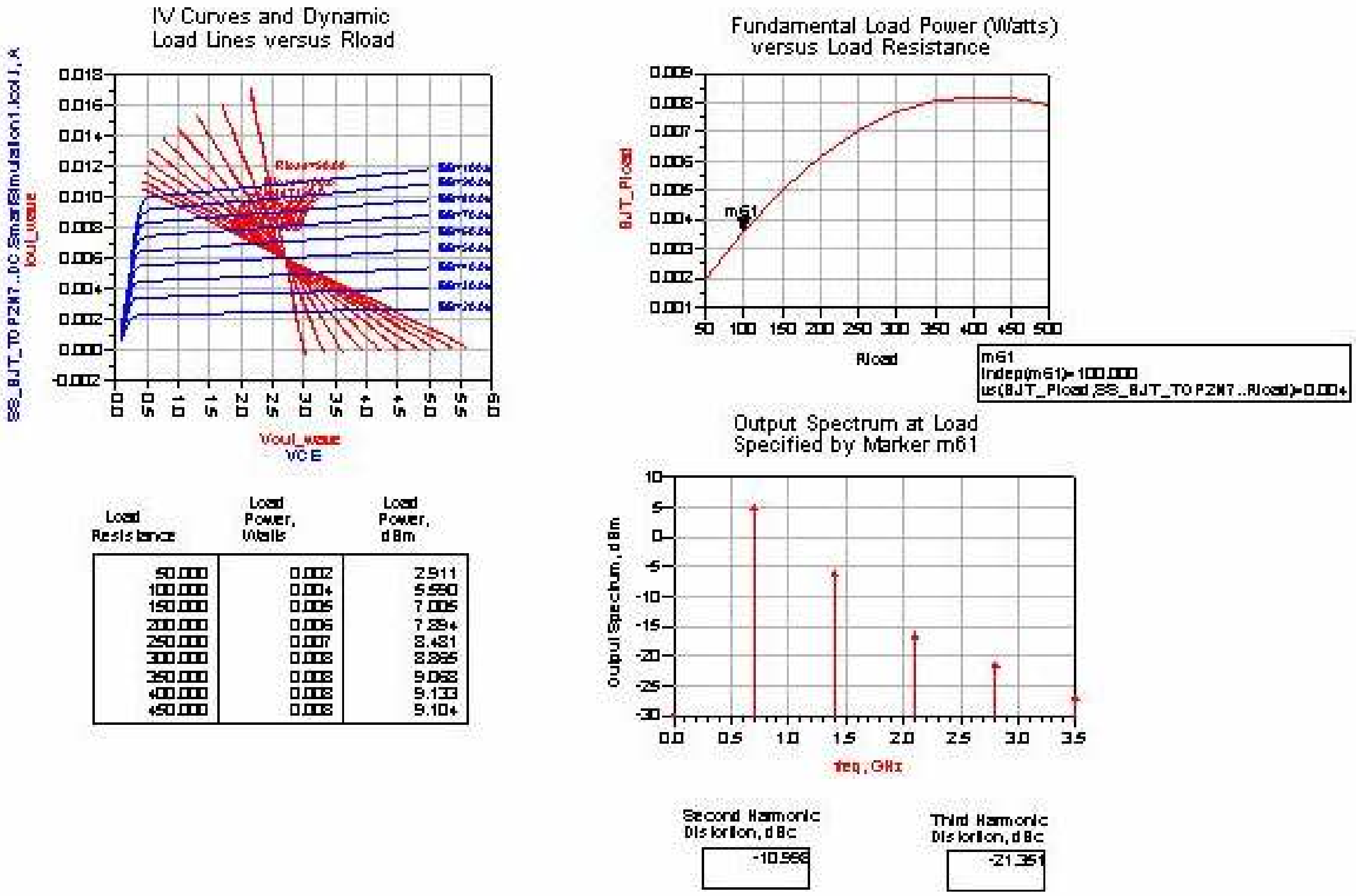


Figure 3.10: Simulation result for IV curve tracer



Load Resistance	Load Power, Watts	Load Power, dBm
50.000	0.002	2.911
100.000	0.004	5.590
150.000	0.005	7.005
200.000	0.006	7.894
250.000	0.007	8.481
300.000	0.008	8.865
350.000	0.008	9.068
400.000	0.008	9.133
450.000	0.008	9.104

Figure 3.11: Simulation result for Power spectrum ,Harmonic distortion Vs Load

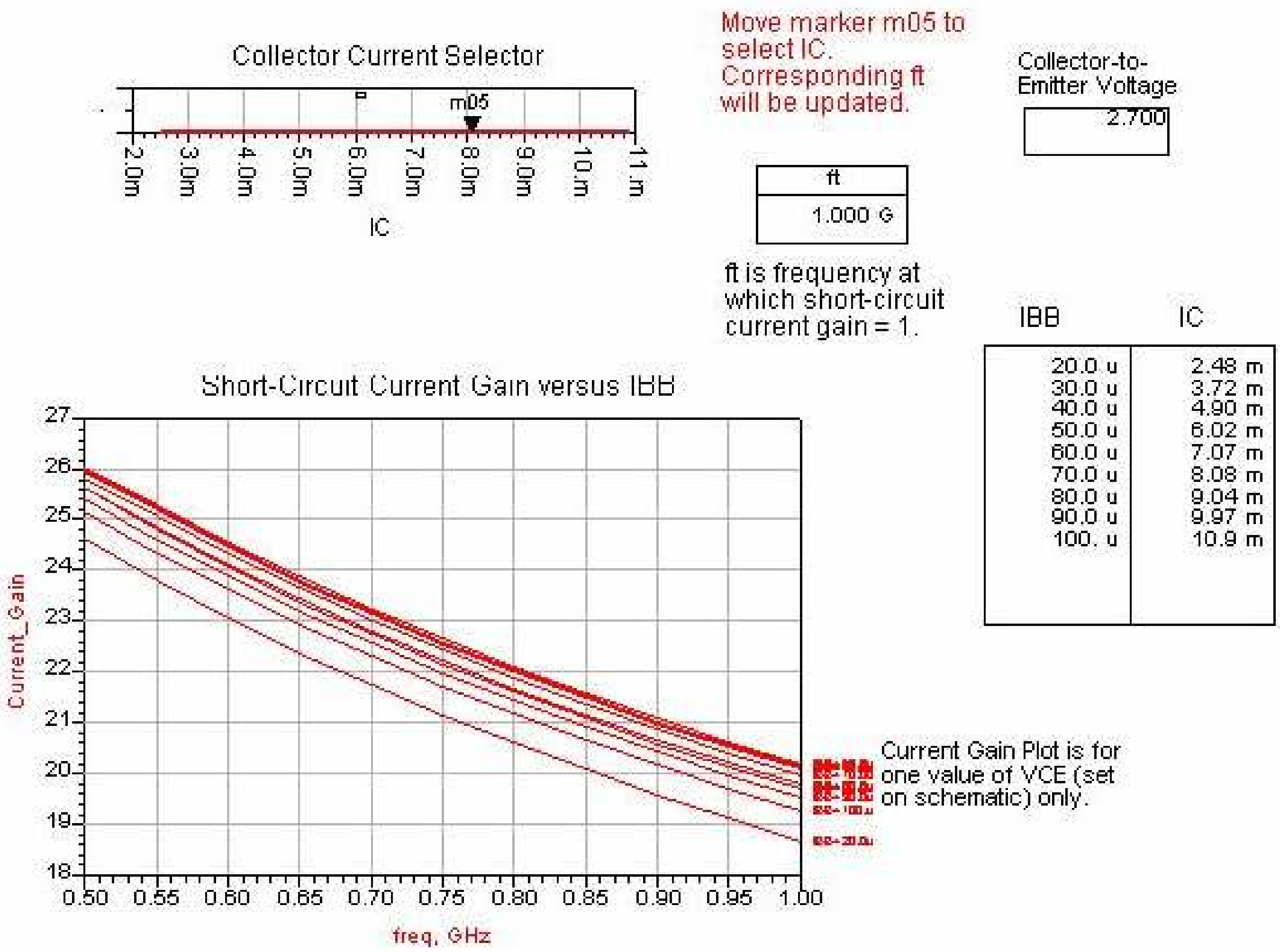


Figure 3.12: Simulation result for Unity current gain frequency Vs bias

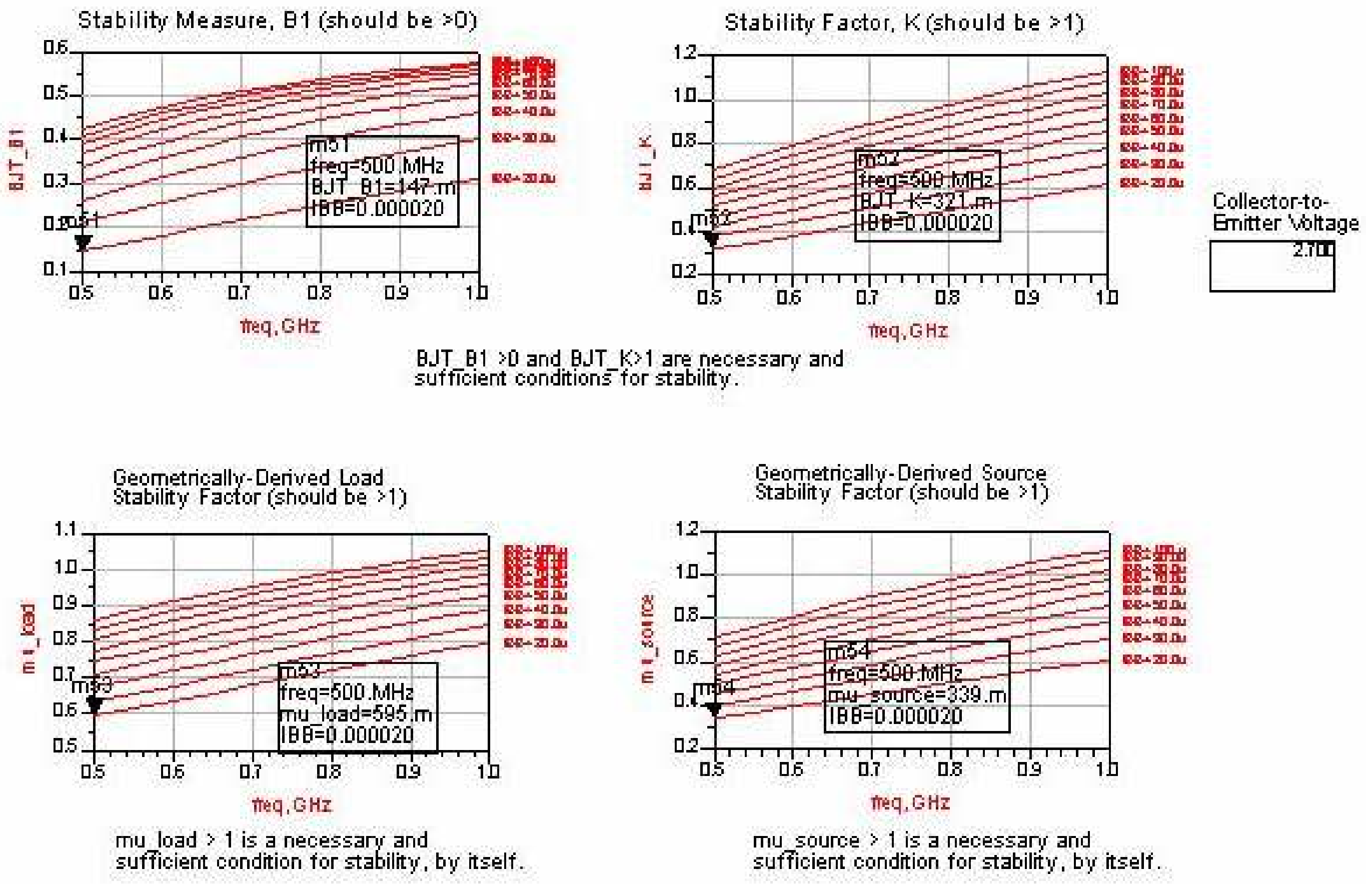


Figure 3.13: Simulation result for stability Vs Frequency

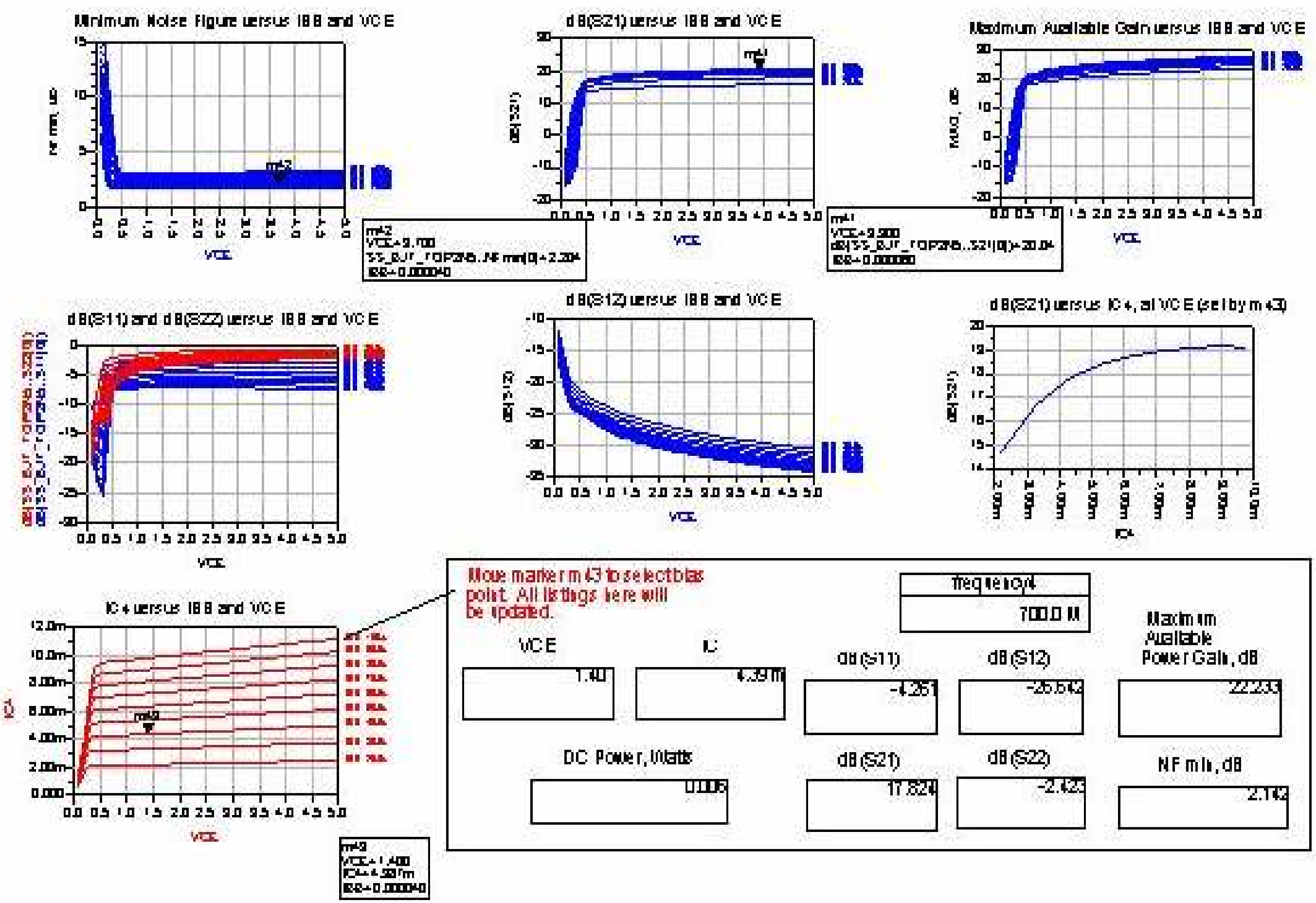


Figure 3.14: Simulation result for stability Vs Bias

Chapter 4

CONCLUSION

4.1 CONCLUSION

Before this report draws to a close, the general conclusions that emerge out from this work and presented in the literature are highlighted .These conclusions are arrived at based on the design of a low noise amplifier using bipolar junction transistor.

1. A power gain of 10 or more is desirable, since this will make small noise figure.

2. It is desirable to have a lower noise figure and yet without sacrificing any gain. But an increase in VSWR at both the input and the out put of the amplifier may be accepted.

3. It is found that the value of VSWR(in) value is either very slowly or remains almost same with the increase in the coefficients of S-Matrix.

4. The value of VSWR(out),however ,increases to a large value when the coefficients of S-Matrix are increased.

5. The gain of the amplifier, although their trend is of increasing in nature but changes a very little.

6. The noise figure also shows a similar increasing trend as in the gain with little more variation, however no variation is found with S_{21}.

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